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THERMAL EXPANSION OF ALPHA-ZIRCONIUM SINGLE CRYSTALS

by

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ABSTRACT

Single crystals of alpha zirconium have been used to measure, between room temperature and 860°C, the thermal expansions in the two principal crystallographic directions of the hexagonal-close-packed structure. The measurements were made with a dilatation interferometer. Equations which represent the lengths along the a and c axes and volume as functions of temperature are as follows:

$$L_{t°C}(\perp \text{ to c axis}) = L_{0°C}(1 + 5.145 \times 10^{-6} t);$$

$$L_{t°C}(\parallel \text{ to c axis}) = L_{0°C}(1 + 9.213 \times 10^{-6} t - 6.385 \times 10^{-9} t^2 + 18.491 \times 10^{-12} t^3 - 9.856 \times 10^{-15} t^4);$$

$$V_{t°C} = V_{0°C}(1 + 19.756 \times 10^{-6} t - 7.023 \times 10^{-9} t^2 + 19.146 \times 10^{-12} t^3 - 9.890 \times 10^{-15} t^4) \quad .$$

These results do not differ greatly from previous thermal expansion data which were obtained by measurements of the lattice constants at different temperatures. But the dilatation data are for the entire temperature range, whereas the lattice-constant data were for two different ranges. The dilatation data indicate that the lattice constants at the higher temperature were subject to error because of contamination by gaseous impurities.

I. INTRODUCTION

In most cases, to evaluate properties of a material correctly from measurements as a function of temperature, it is necessary to know the accompanying dimensional changes of the specimens. This type of problem has arisen in connection with studies of the elastic moduli in single crystals of alpha zirconium. Since the low-temperature modification of zirconium has an anisotropic structure (close-packed-hexagonal), it is not possible to determine its correct thermal expansion behavior by simple linear measurements of polycrystalline materials. Thermal-expansion data as a function of crystallographic direction can be obtained if the temperature dependence of the two lattice constants are known. A second method of obtaining this information is by dilatometric measurements of single crystals.

Two sets of data are available for the temperature variation of the lattice constants, a and c, over portions of the temperature range wherein alpha zirconium is stable (-273 to 862°C). Russell(1) has reported data for hafnium-free zirconium from room temperature up to approximately 580°C. Above this temperature the grain size in his samples became large enough to give spotty X-ray diffraction lines. Also, at this temperature, the contrast of the lines on the film had decreased sufficiently so that the number of measurable lines was reduced. Skinner and Johnston(2) have given lattice constants for zirconium, which contained approximately 1 a/o hafnium and other impurities, at room temperature and between 677°C and the $\alpha \rightleftharpoons \beta$ transformation temperature. In reporting values for a and c, they applied two corrections: 1) that for chemical impurities in the material and 2) that for the change in room-temperature lattice constants which resulted from absorption, at the higher temperatures, of gaseous impurities (chiefly nitrogen and oxygen). When combined plots are made for the two sets of data, considerable uncertainty remains as to the temperature dependence of the lattice constants.

No information is available in the literature with regard to the thermal expansion of single crystals of alpha zirconium. This type of data has certain advantages over lattice-constant measurements. Dilatation measurements are obtained as a continuous, or almost continuous, function of temperature, whereas lattice constants are measured at particular temperatures. Consequently, errors in the former type of data are more systematic than in the latter. Also, dilatometric tests take considerably less time than a series of high-precision lattice-constant determinations which is required to provide the same information. This is particularly advantageous in the case of zirconium because of its affinity for oxygen and nitrogen. Along the same line of consideration is the fact that X-ray diffraction methods for determining lattice constants utilize a relatively thin surface layer of the sample, the portion first contaminated by gaseous impurities. Dilatometric measurements, on the other hand, are obtained from bulk specimens, and any contamination would be expected to affect the results to a lesser extent.

In view of the need for thermal-expansion data as a function of crystallographic orientation in alpha zirconium, particularly at the higher temperatures, dilatation measurements have been obtained from single-crystal specimens. This report describes the results of the study.

II. EXPERIMENTAL

Most dilatometric techniques involve the measurement of the differential between a sample of unknown thermal expansion and a material of known thermal expansion. One exception is the method, which was selected for the present study, based upon interference patterns which are created by reflection of light from two optically flat surfaces. Since the reflecting surfaces are separated by the ends of the specimens, no other material enters into the measurement, provided the tests are performed in vacuum. It suffices to record the coincidence of interference fringes with a reference point as a function of temperature, and, knowing the wavelength of the illuminating light plus the original lengths of the samples, the thermal expansion can be calculated. The principal source of error is that associated with temperature measurement.

A. Specimen Preparation

Since the primary objective was to provide thermal-expansion data for calculation of elastic moduli from measurements of ultrasonic wave velocities in alpha-zirconium single crystals, the single crystals used for the dilatation tests were prepared from similar, and in some cases the same, material as those used in the ultrasonic studies. To prepare the single crystals, Westinghouse, hafnium-free, grade I, iodide crystal bar zirconium was subjected to the grain-growth treatment described by Langeron and Lehr.[3]

A typical analysis of the nongaseous impurities in the material is given in Table I. Normal nitrogen contents are about 10 ppm by weight. Oxygen contents as high as 200 ppm have been reported for crystal bar zirconium, but none of the present materials have given analyses as high as that, and one analysis was as low as 15 ppm. Hydrogen content is not significant since the crystals were given a dehydrogenation treatment

Table I

TYPICAL ANALYSIS OF WESTINGHOUSE GRADE I
CRYSTAL BAR ZIRCONIUM

Element	Content (ppm by weight)	Element	Content (ppm by weight)	Element	Content (ppm by weight)
Ag	<1	Fe	400	Sb	<1
Al	<10	Hf	<500	Si	150
As	<50	K	<20	Sn	<5
B	<0.1	Li	<1	Sr	<100
Ba	<20	Mg	3	Ta	<500
Be	<1	Mn	<1	Ti	<20
Ca	<100	Na	7	W	<100
Cd	<20	Nb	<100	V	<20
Co	<1	Ni	7	Zn	<20
Cr	30	P	<50		
Cu	7	Pb	8		

prior to dilatation testing. As shown in Table I, the iron content could be reasonably high. In fact, some of the crystals contained small, isolated areas of second-phase material which was believed to be $ZrFe_2$, but its presence did not appear to affect the thermal expansion behavior, as is discussed below.

Six single crystals, two sets of three each, were used for the dilatometric tests. Three specimens were prepared with their testing directions normal to (00·1) planes and the other three were normal to {10·0} planes. The techniques and equipment for orienting the crystals by Laue back-reflection X-ray photograms, for mounting them parallel to a specific crystallographic plane, and for grinding and lapping them, have been described by Fisher.(4) One flat face of the desired orientation was prepared on each of the crystals, and this face was lapped parallel to the reference surface of the mounting ring. This face served as a reference surface for subsequent operations. Next, a second face was ground and lapped parallel to the first face. Then four other faces were ground on the sides of each crystal to form a truncated pyramid with the original flat surface as the base. After final grinding, each of the side faces was etched in a solution of 7 v/o 48% HF in a 50:50 mixture of concentrated HNO_3 and water, followed by a second etch in 50:50 HNO_3 and water until the gross distortion effects of the grinding were removed.

Since the single crystals contained hydride precipitates, they were then removed from the mounts and subjected to a dehydrogenation treatment. This consisted of heating the samples to 800°C for 24 hr in a vacuum of at least 2×10^{-6} mm Hg. The residual hydrogen content should have been reduced to below 1 ppm by weight. After this treatment no hydride precipitates were detected by metallography. The heat treatment also served to remove the majority of the residual strains from the grinding and lapping operations.

Each of the matching sets of crystals was remounted, one set per ring, with the pyramid base parallel to one end of the ring. The top and bottom surfaces of the pyramids were reground and lapped to create flat, parallel surfaces, as well as nearly equal lengths, for the three crystals of each set. The samples were then removed from the mounting rings in preparation for dilatation testing.

Prior to the tests, the weights and heights of each specimen were measured. Weights were determined with a Gram-atic balance to the nearest 10^{-4} gram. Lengths were measured by a micrometer to the nearest 10^{-4} in., with the specimen and micrometer submerged in a water bath whose temperature was measured to the nearest 0.1°C.

B. Dilatation Interferometer

The basic instrument employed in the dilatometric measurements was a Gaertner interferometer which used mercury vapor green light for illumination. The quartz optical flats and the specimens were heated in a vacuum furnace. The interferometer assembly was similar to the one described by Merritt.(5) The arrangement of the internal components of the furnace is shown schematically in Figure 1. The vacuum was maintained by a system composed of a liquid nitrogen cold trap, a Distillation Products MC500-06 oil diffusion pump, a Distillation Products DS100 oil booster pump, and a Welch Duo-Seal 1397 roughing pump. The quality of the vacuum was monitored with a VG1A ion-tube. A vacuum in the range from 5 x 10^{-5} to 5 x 10^{-7} mm Hg was maintained by this system during the dilatometric runs.

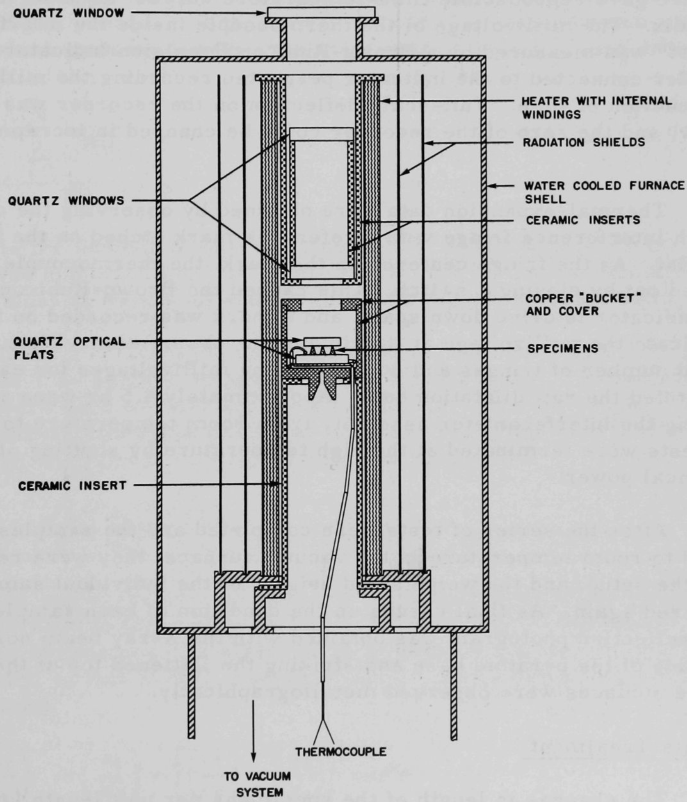

Fig. 1. Schematic Drawing of Dilatation Interferometer

In order to provide a more uniform temperature around the optical flats and the specimens, the interferometer "bucket" and cover were made of copper. To reduce the temperature differential between the chromel-alumel thermocouple bead and the specimens, the thermocouple junction was placed as closely as possible to the specimens and in contact with the upper surface of the lower optical flat. The thermocouple was brought into the "bucket" through a hole drilled in the bottom and near the outer edge. The two wires of the thermocouple were separated, fitted around the lower optical flat, and then brought together again at a position 180° from the drilled hole. The slight spring action of the wires served to maintain the thermocouple bead in contact with the optical flat.

A motor-driven cam which rotated an adjustable autotransformer provided electrical power to the furnace. This system of controlling temperature gave reproducible time-temperature curves, as described in the Appendix. The millivoltage of the thermocouple inside the interferometer "bucket" was measured by a Brown-Rubicon Precision Indicator. A "slave" recorder connected to the indicator permitted recording the millivoltage as a function of time. Full-scale deflection on the recorder was equal to 1.1 mv, and the zero of the recorder could be changed in increments of 1 mv.

Thermal-expansion data were obtained by observing the coincidence of each interference fringe with a reference mark etched on the lower optical flat. As the fringe centered on the mark, the thermocouple was shorted out by closing a switch. This caused the Brown-Rubicon Precision Indicator to drive down scale, and a mark was recorded on the chart to indicate the millivoltage at the time of the observation. Thus, the observed number of fringes and corresponding millivoltages for each fringe constituted the raw dilatation data. Approximately 4.5 hr were required to bring the interferometer assembly from room temperature to 860°C. The tests were terminated at the high temperature by shutting off the electrical power.

After the series of tests were completed and the samples were cooled to room temperature in the vacuum furnace, they were removed from the setup, and the weights and heights of the individual samples were measured again. As final checks on the condition of each sample, a Laue back-reflection photogram was obtained with the X-ray beam normal to the plane of the pyramid base and striking the flattened top of the pyramid, and the surfaces were observed metallographically.

C. Data Treatment

The changes in length of the specimens per unit length (ΔL) were computed from the number of the interference fringe (n), which coincided with the reference mark, according to the equation

$$\Delta L = n\lambda/2L_0 \quad , \tag{1}$$

where λ is the wavelength of the illumination (5461 Å) and L_0 is the original length of the specimens. The corresponding thermocouple millivoltage was corrected according to the procedure given in the Appendix and converted to temperature in °C. These data then were used to construct plots of length change of the specimen as a function of temperature.

The data were submitted to an IBM-704 computer (program ANE-205) for least-square-polynomial fitting to equations of the type

$$Y = k_0 + k_1 t + k_2 t^2 \ldots\ldots + k_p t^p \quad , \tag{2}$$

where Y represents the dimension at temperature t°C, the k's are constants, and p is the order of the polynomial. In all cases equations were determined for p equal to 1 through 4. The best fit was selected as the equation which gave the lowest standard deviation (σ) according to the equation

$$\sigma = \frac{\sum_{i=1}^{N} |R_i|}{N} \sqrt{\frac{\pi}{2}} \quad , \tag{3}$$

where N is the number of observations and R_i is the difference between the observed and calculated i^{th} datum point. The lattice constants reported by Russell were also submitted to this type of equation fitting.

Values of Y from the various equations were calculated for given intervals of t by another computer program (ANE-102). The resulting numbers were used to calculate the temperature variation of other factors for the alpha-zirconium lattice. The ratio of the volume at t°C to that at 0°C (namely, V_t/V_0) was calculated from the relation[6]

$$\frac{V_t}{V_0} = \frac{\sqrt{3}}{2} \left(\frac{a_t}{a_0}\right)^2 \left(\frac{c_t}{c_0}\right) \quad , \tag{4}$$

where a_t/a_0 and c_t/c_0 are the ratios of the lattice dimensions perpendicular and parallel to the c axis at t°C to those at 0°C, respectively. The ratio of the dimensions of a direction at an angle θ degrees from the c axis at t°C to that at 0°C, namely, $(L_t/L_0)_{\theta°}$, was calculated according to the equation

$$\left(\frac{L_t}{L_0}\right)_{\theta°} = \left(\frac{a_t}{a_0}\right) + \left[\left(\frac{c_t}{c_0}\right) - \left(\frac{a_t}{a_0}\right)\right] \cos^2\theta \quad . \tag{5}$$

Also, equations as a function of temperature were determined for the lattice constants a and c, axial ratio, volume per atom (V/atom), and the distance of closest approach (d_1) from the expansion data and a set of room-temperature lattice constants given in the literature. The relationships used to evaluate the latter two dimensions were:

$$V/\text{atom} = \frac{\sqrt{3}}{4} a^2 c \tag{6}$$

and

$$d_1 = \left(\frac{a^2}{3} + \frac{c^2}{4}\right)^{1/2} \quad . \tag{7}$$

From the values of a, c, and V/atom at the different temperatures, it was possible to determine equations as functions of temperature which represented true thermal expansion coefficients α_Y as defined by the relationship

$$\alpha_Y = \frac{1}{Y} \frac{dY}{dt} \quad . \tag{8}$$

Values of ln Y versus t were fitted to polynomial equations, as described above, to give

$$\ln Y = k_0 + k_1 t + k_2 t^2 + \ldots . + k_p t^p \tag{9}$$

Differentiation of this type of equation gave

$$\alpha_Y = \frac{1}{Y} \frac{dY}{dt} = k_1 + \frac{k_2}{2} t + \ldots . + \frac{k_p}{p} t^{(p-1)} \quad .$$

III. RESULTS AND DISCUSSION

A. Dilatation Data

The dimensions and weights of the single-crystal specimens before and after dilatation testing are given in Table II. Orientations of the pyramidal axes were within one-half degree of the desired orientations. No significant variations were noted between the initial and final measurements of pyramid height and weight. The differences that were observed were in the fourth decimal places, which were obtained, directly or indirectly, by reading verniers on both the balance and the micrometer. Since the crystals were heated to 860°C several times without significant weight or dimensional changes and since no systematic drifts of the expansion curves were observed with the number of runs, the vacuum furnace provided good protection for the samples against contamination by gaseous impurities. Also, the samples were not deformed by creep at the higher temperatures as a result of the load applied by the upper optical flat.

Table II

DIMENSIONS AND WEIGHTS OF ALPHA-ZIRCONIUM SINGLE CRYSTALS USED FOR DILATATION TESTS

Specimen Identity	Weight (gm)		Pyramid Dimensions*(cm)							
			Height				Base		Top	
			Initial		Final					
	Initial	Final	Dimension	Measuring Temperature (°C)	Dimension	Measuring Temperature (°C)	Edges 1 and 3	Edges 2 and 4	Edges 1 and 3	Edges 2 and 4
(00·1) - Runs 1 and 2										
No. 1	0.3698	0.3697	0.6015	15.5	0.6015	15.5				
No. 2	0.3821	0.3822	0.6017	15.5	0.6017	15.5				
No. 3	0.3861	0.3862	0.6017	15.5	0.6017	15.5				
(00·1) - Runs 3 and 4										
No. 1	0.3697	0.3697	0.6015	15.5	0.6015	15.7				
No. 2	0.3822	0.3822	0.6017	15.5	0.6015	15.7				
No. 3	0.3862	0.3862	0.6017	15.5	0.6017	15.7				
(00·1) - Runs 5, 6 and 7										
No. 1	0.3697	0.3697	0.6015	15.7	0.6015	15.6	0.508	0.480	0.114	0.089
No. 2	0.3822	0.3821	0.6015	15.7	0.6015	15.6	0.505	0.483	0.114	0.094
No. 3	0.3862	0.3863	0.6017	15.7	0.6015	15.6	0.488	0.480	0.147	0.094
{10·0} - Runs 1 and 2										
No. 4	0.2374	0.2374	0.5956	15.7	0.5954	15.5				
No. 5	0.2457	0.2457	0.5954	15.7	0.5951	15.5				
No. 6	0.2738	0.2738	0.5956	15.7	0.5954	15.5				
{10·0} - Run 3										
No. 4	0.2374	0.2374	0.5954	15.5	0.5954	15.5				
No. 5	0.2457	0.2457	0.5951	15.5	0.5951	15.5				
No. 6	0.2738	0.2738	0.5954	15.5	0.5954	15.5				
{10·0} - Runs 4, 5, 6 and 7										
No. 4	0.2210	0.2211	--**	--	0.5750	15.5	0.386	0.343	0.132	0.089
No. 5	0.2284	0.2284	--	--	0.5750	15.5	0.384	0.348	0.135	0.094
No. 6	0.2459	0.2458	--	--	0.5745	15.5	0.391	0.368	0.122	0.117

*All dimensional measurements were made in inches and converted to centimeters. Dimensions of the pyramid base and top were taken after completion of thermal-expansion measurements.

**Pyramid heights were not measured after regrinding and before thermal-expansion measurements.

1. Expansion Parallel to the c Axis. Seven dilatation tests were run up to 860°C for each set of specimens. Four runs of the samples whose directions of testing were parallel to the c axis [the (00·1) samples] were carried out with the thermocouple bead inserted into a blind hole in the bottom of the copper "bucket." This arrangement gave too large a differential between the true and measured temperatures of the samples. The remaining three runs were made with the thermocouple bead located in contact with the lower optical flat, as described above. Data from the first of the latter runs were neglected on the assumption that the samples had to be "seated in" by heating them through the entire temperature cycle. Table III gives the change in length of the samples, based upon unit length of the original samples, and the observed temperatures, which were corrected according to the method given in the Appendix, for the last two runs. These data are plotted in Figure 2; half of the points for each set of data

Table III

CHANGE IN LENGTH PARALLEL TO THE c AXIS OF ALPHA ZIRCONIUM - (00·1) SPECIMENS

Fringe No.	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (°C)		Fringe No.	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (°C)		Fringe No.	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (°C)	
		Run No. 6	Run No. 7			Run No. 6	Run No. 7			Run No. 6	Run No. 7
0	0.00000	16.3	18.4	70	31.76432	370.8	373.8	140	63.52864	638.9	643.0
1	0.45378	20.3	21.7	71	32.21810	375.2	378.0	141	63.98242	642.2	646.5
2	0.90755	24.4	26.3	72	32.67187	379.8	382.7	142	64.43619	645.6	649.9
3	1.36133	28.6	31.0	73	33.12565	384.1	387.0	143	64.88997	649.0	653.2
4	1.81510	33.2	36.3	74	33.57942	388.6	391.4	144	65.34374	651.9	656.6
5	2.26888	38.0	41.6	75	34.03320	392.6	395.9	145	65.79752	655.6	660.0
6	2.72266	43.3	47.0	76	34.48698	397.1	400.2	146	66.25130	658.7	663.2
7	3.17643	48.4	52.2	77	34.94075	401.3	-	147	66.70507	662.0	666.5
8	3.63021	53.6	57.2	78	35.39453	405.8	408.6	148	67.15885	665.3	670.0
9	4.08398	58.6	62.9	79	35.84830	409.9	413.1	149	67.61262	668.4	673.1
10	4.53776	63.9	68.6	80	36.30208	414.0	417.2	150	68.06640	671.6	676.2
11	4.99154	69.2	73.8	81	36.75586	418.3	421.4	151	68.52018	675.1	679.5
12	5.44531	74.6	78.9	82	37.20963	422.5	425.6	152	68.97395	678.3	682.7
13	5.89909	79.8	84.2	83	37.66341	426.9	430.0	153	69.42773	681.4	685.9
14	6.35286	85.2	-	84	38.11718	431.0	434.8	154	69.88150	684.9	689.2
15	6.80664	90.4	95.0	85	38.57096	435.4	438.2	155	70.33528	687.9	692.5
16	7.26042	96.0	100.5	86	39.02474	439.4	442.5	156	70.78906	691.1	695.7
17	7.71419	101.4	105.9	87	39.47851	443.4	446.6	157	71.24283	694.4	699.0
18	8.16797	-	111.6	88	39.93229	447.6	450.8	158	71.69661	697.6	702.4
19	8.62174	111.9	117.1	89	40.38606	451.6	454.7	159	72.15038	700.8	705.7
20	9.07552	117.6	122.4	90	40.83984	455.7	458.7	160	72.60416	704.2	708.9
21	9.52930	122.9	127.9	91	41.29362	459.7	462.9	161	73.05794	707.4	712.2
22	9.98307	128.3	133.3	92	41.74739	463.8	466.8	162	73.51171	710.6	-
23	10.43685	133.9	138.6	93	42.20117	467.8	470.9	163	73.96549	714.0	718.8
24	10.89062	139.3	143.9	94	42.65494	471.8	474.9	164	74.41926	717.2	722.0
25	11.34440	144.6	149.3	95	43.10872	475.8	478.8	165	74.87304	720.5	725.4
26	11.79818	149.9	154.6	96	43.56250	479.9	482.9	166	75.32682	723.7	728.5
27	12.25195	155.2	160.0	97	44.01627	484.0	486.7	167	75.78059	726.9	731.8
28	12.70573	160.5	165.2	98	44.47005	487.6	490.8	168	76.23437	730.3	735.2
29	13.15950	165.8	170.3	99	44.92382	491.6	494.7	169	76.68814	733.3	738.3
30	13.61328	-	175.6	100	45.37760	495.4	498.6	170	77.14192	736.6	741.6
31	14.06706	176.6	180.9	101	45.83138	499.2	502.4	171	77.59570	740.0	744.9
32	14.52083	181.4	186.2	102	46.28515	503.3	506.4	172	78.04947	743.2	748.0
33	14.97461	187.2	191.2	103	46.73893	507.1	510.3	173	78.50325	746.5	751.3
34	15.42838	192.6	196.6	104	47.19270	511.0	514.2	174	78.95702	749.6	-
35	15.88216	198.0	201.8	105	47.64648	514.8	518.0	175	79.41080	753.0	757.5
36	16.33594	203.3	207.1	106	48.10026	518.6	521.9	176	79.86458	756.2	760.5
37	16.78971	208.6	212.1	107	48.55403	522.1	525.7	177	80.31835	759.6	764.0
38	17.24349	213.8	217.5	108	49.00781	526.0	529.4	178	80.77213	762.8	766.9
39	17.69726	219.3	222.8	109	49.46158	529.8	533.3	179	81.22590	765.9	770.3
40	18.15104	224.8	227.9	110	49.91536	533.6	537.0	180	81.67968	769.3	773.2
41	18.60482	230.0	233.2	111	50.36914	537.4	540.8	181	82.13346	772.3	776.5
42	19.05859	235.5	238.2	112	50.82291	541.1	544.5	182	82.58723	775.6	779.7
43	19.51237	240.2	243.1	113	51.27669	544.8	548.2	183	83.04101	779.0	783.0
44	19.96614	245.5	248.6	114	51.73046	548.3	551.8	184	83.49478	782.1	786.2
45	20.41992	250.6	253.4	115	52.18424	551.9	555.6	185	83.94856	785.4	789.2
46	20.87370	255.8	258.6	116	52.63802	555.7	559.3	186	84.40234	788.3	792.5
47	21.32747	260.8	263.5	117	53.09179	559.2	562.7	187	84.85611	791.5	795.9
48	21.78125	266.0	268.7	118	53.54557	562.8	566.4	188	85.30989	795.0	798.9
49	22.23502	271.1	273.8	119	53.99934	566.4	570.0	189	85.76366	798.1	801.9
50	22.68880	276.0	278.8	120	54.45312	570.0	573.8	190	86.21744	801.1	805.2
51	23.14258	281.2	283.7	121	54.90690	573.6	577.2	191	86.67122	804.4	808.2
52	23.59635	286.2	288.9	122	55.36067	577.1	580.7	192	87.12499	807.7	811.5
53	24.05013	291.2	293.6	123	55.81445	580.5	584.2	193	87.57877	811.0	814.8
54	24.50390	-	298.4	124	56.26822	584.1	588.0	194	88.03254	814.1	817.7
55	24.95768	301.0	303.3	125	56,72200	587.7	591.5	195	88.48632	817.3	820.8
56	25.41146	305.8	308.3	126	57.17578	591.1	594.9	196	88.94010	820.2	824.0
57	25.86523	310.5	313.0	127	57.62955	594.7	598.3	197	89.39387	823.6	827.1
58	26.31901	315.4	317.7	128	58.08333	598.0	601.8	198	89.84765	826.6	830.0
59	26.77278	320.2	-	129	58.53710	601.4	605.4	199	90.30142	829.7	833.3
60	27.22656	325.0	327.5	130	58.99088	604.8	609.1	200	90.75520	833.0	836.5
61	27.68034	329.6	332.0	131	59.44466	608.2	612.3	201	91.20898	836.3	839.4
62	28.13411	334.6	337.1	132	59.89843	611.8	615.7	202	91.66275	839.3	842.3
63	28.58789	339.0	341.4	133	60.35221	615.0	619.2	203	92.11653	842.3	845.6
64	29.04166	343.8	345.9	134	60.80598	618.4	622.7	204	92.57030	845.6	849.6
65	29.49544	348.2	350.7	135	61.25976	621.6	626.1	205	93.02408	848.7	852.0
66	29.94922	352.8	355.4	136	61.71354	625.1	629.5	206	93.47786	852.0	854.8
67	30.40299	357.2	360.0	137	62.16731	628.8	632.9	207	93.93163	855.2	857.7
68	30.85677	361.8	364.6	138	62.62109	632.0	636.3	208	94.38541	858.2	860.8
69	31.31054	366.3	369.0	139	63.07486	635.4	639.7	209	94.83918	861.4	

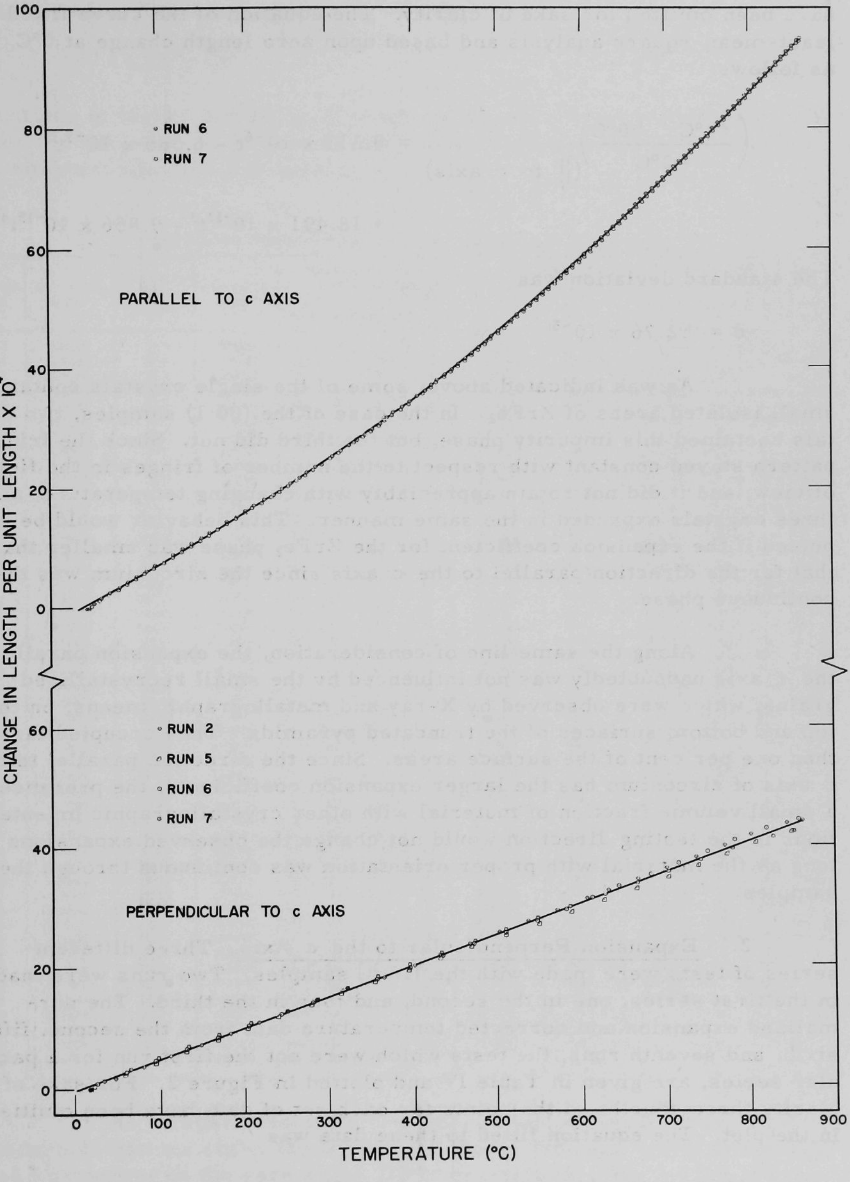

Fig. 2. Change in Length with Temperature for the Two Principal Crystallographic Directions of Alpha Zirconium

have been omitted for sake of clarity. The equation of the curve fitted by least-mean-square analysis and based upon zero length change at 0°C, is as follows:

$$\left(\frac{L_{t°C} - L_{0°C}}{L_{0°C}}\right)_{(\parallel \text{ to c axis})} = 9.213 \times 10^{-6}t - 6.385 \times 10^{-9}t^2 + 18.491 \times 10^{-12}t^3 - 9.856 \times 10^{-15}t^4 \quad .$$

The standard deviation was

$$\sigma = \pm 2.76 \times 10^{-5} \quad .$$

As was indicated above, some of the single crystals contained small isolated areas of $ZrFe_2$. In the case of the (00·1) samples, two crystals contained this impurity phase, but the third did not. Since the fringe pattern stayed constant with respect to the number of fringes in the field of view, and it did not rotate appreciably with changing temperature, all three crystals expanded in the same manner. This behavior would be expected if the expansion coefficient for the $ZrFe_2$ phase was smaller than that for the direction parallel to the c axis since the zirconium was the continuous phase.

Along the same line of consideration, the expansion parallel to the c axis undoubtedly was not influenced by the small recrystallized grains, which were observed by X-ray and metallographic means, on the top and bottom surfaces of the truncated pyramids. They occupied less than one per cent of the surface areas. Since the direction parallel to the c axis of zirconium has the larger expansion coefficients, the presence of a small volume fraction of material with other crystallographic orientations in the testing direction would not change the observed expansions as long as the material with proper orientation was continuous through the samples.

2. Expansion Perpendicular to the c Axis. Three different series of tests were made with the {10·0} samples. Two runs were made in the first series, one in the second, and four in the third. The normalized expansion and corrected temperature data from the second, fifth, sixth, and seventh runs, the tests which were not the first run for a particular series, are given in Table IV and plotted in Figure 2. For sake of clarity three-fourths of the points for each set of data have been omitted in the plot. The equation fitted to these data was

$$\left(\frac{L_{t°C} - L_{0°C}}{L_{0°C}}\right)_{(\perp \text{ to c axis})} = 5.145 \times 10^{-6}t$$

and the standard deviation was

$$\sigma = \pm 4.99 \times 10^{-5} \quad .$$

Equations of higher power, up through the fourth, fit the data equally well. Since the variation in σ was not more than 1% for the different data fits, the simplest equation was selected.

Table IV

CHANGE IN LENGTH PERPENDICULAR TO THE c AXIS OF ALPHA ZIRCONIUM - {10·0} SPECIMENS

Fringe No.	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (^oC) Run No. 2	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (^oC)			Fringe No.	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (^oC) Run No. 2	$\Delta L = n\lambda/2L_0$ (x 10^4)	Temperature (^oC)		
				Run No. 5	Run No. 6	Run No. 7					Run No. 5	Run No. 6	Run No. 7
0	0.00000	17.5	0.00000	18.6	18.0	18.3	49	22.46263	439.6	23.26628	467.7	462.1	452.7
1	0.45842	24.0	0.47482	21.8	18.4	20.8	50	22.92105	447.2	23.74110	--	471.0	462.6
2	0.91684	31.2	0.94964	31.1	26.1	26.7	51	23.37947	457.3	24.21592	488.3	--	472.7
3	1.37526	39.4	1.42447	40.9	34.2	33.9	52	23.83789	465.9	24.69074	498.2	489.7	481.7
4	1.83368	47.3	1.89929	50.3	42.9	40.6	53	24.29631	475.2	25.16557	508.2	499.1	490.9
5	2.29210	55.7	2.37111	59.2	51.7	48.2	54	24.75473	484.0	25.64039	518.5	508.3	499.3
6	2.75053	66.4	2.84893	68.2	59.9	56.3	55	25.21316	492.4	26.11521	527.9	518.2	508.4
7	3.20895	72.2	3.32375	76.5	68.4	64.4	56	25.67158	500.9	26.59003	537.7	526.9	519.2
8	3.66737	80.6	3.79858	85.8	77.0	72.4	57	26.13000	509.6	27.06485	547.6	536.6	526.8
9	4.12579	88.9	4.27340	95.1	85.5	80.8	58	26.58842	518.2	27.53968	557.6	545.7	537.0
10	4.58421	97.4	4.74822	104.0	94.5	89.7	59	27.04684	526.7	28.01450	567.6	554.2	545.9
11	5.04263	105.7	5.22304	112.3	103.4	98.4	60	27.50526	534.7	28.48932	576.6	562.6	556.0
12	5.50105	113.4	5.69786	122.6	111.8	108.1	61	27.96368	542.8	28.96414	586.0	571.0	565.9
13	5.95947	121.8	6.17269	131.3	122.0	116.8	62	28.42210	551.2	29.43896	595.7	579.5	574.5
14	6.41789	130.6	6.64751	140.3	131.4	125.5	63	28.88052	559.4	29.91379	605.3	587.5	584.5
15	6.87632	139.6	7.12233	150.5	140.9	134.0	64	29.33894	568.8	30.38861	615.1	595.4	594.5
16	7.33474	148.0	7.59715	--	150.6	143.1	65	29.79736	577.8	30.86343	624.4	603.4	603.6
17	7.79316	157.0	8.07197	167.5	159.2	151.5	66	30.25579	586.6	31.33825	634.1	610.7	612.4
18	8.25158	165.4	8.54680	176.6	168.5	160.4	67	30.71421	595.4	31.81307	643.9	618.6	625.8
19	8.71000	174.1	9.02162	185.9	177.7	169.2	68	31.17263	604.3	32.28790	653.7	626.5	635.1
20	9.16842	182.8	9.49644	194.7	187.1	181.5	69	31.63105	612.5	32.76272	663.0	634.2	642.6
21	9.62684	189.2	9.97126	203.9	196.4	190.8	70	32.08947	621.4	33.23754	672.4	642.0	653.6
22	10.08526	199.9	10.44608	213.3	205.1	200.2	71	32.54789	629.8	33.71236	682.2	650.1	664.0
23	10.54368	208.1	10.92091	221.9	214.4	208.2	72	33.00631	638.6	34.18718	691.2	662.8	673.9
24	11.00210	216.8	11.39573	230.6	223.6	219.8	73	33.46473	647.2	34.66201	700.5	671.4	682.7
25	11.46052	224.8	11.87055	239.9	233.2	230.0	74	33.92315	655.9	35.13683	709.7	679.8	691.5
26	11.91895	233.6	12.34537	249.4	242.4	239.4	75	34.38158	664.8	35.61165	718.9	688.3	700.7
27	12.37737	242.2	12.82019	259.0	252.1	249.4	76	34.84000	673.2	36.08647	727.4	696.5	709.6
28	12.83579	250.6	13.29502	268.2	260.8	258.6	77	35.29842	681.6	36.56129	736.1	704.6	719.6
29	13.29421	259.3	13.76984	278.0	270.3	268.1	78	35.75684	690.4	37.03612	744.8	713.0	729.0
30	13.75263	267.6	14.24466	286.8	279.4	277.0	79	36.21526	698.8	37.51094	753.3	721.3	739.1
31	14.21105	275.8	14.71948	296.5	289.0	285.9	80	36.67368	707.9	37.98576	761.7	729.6	748.2
32	14.66947	284.3	15.19430	305.9	298.5	294.8	81	37.13210	716.5	38.46058	769.9	737.6	758.0
33	15.12789	292.6	15.66913	315.6	308.1	303.2	82	37.59052	725.1	38.93540	778.5	746.8	767.1
34	15.58631	301.3	16.14395	325.0	317.5	313.0	83	38.04894	733.6	39.41023	786.9	755.1	777.0
35	16.04474	310.1	16.61877	334.3	327.3	321.5	84	38.50736	742.3	39.88505	795.5	763.1	785.6
36	16.50316	319.8	17.09359	343.9	334.3	330.2	85	38.96578	750.9	40.35987	805.2	771.4	796.6
37	16.96158	333.6	17.56841	352.8	346.2	339.4	86	39.42421	759.4	40.83469	815.9	779.5	813.8
38	17.42000	344.8	18.04324	362.8	356.2	348.2	87	39.88263	767.6	41.30951	825.5	788.2	826.5
39	17.87842	353.6	18.51806	372.3	365.7	357.0	88	40.34105	775.7	41.78434	836.0	797.0	837.7
40	18.33684	363.1	18.99288	382.2	375.7	367.4	89	40.79947	784.2	42.25916	848.0	807.5	851.1
41	18.79526	371.3	19.46770	391.6	385.0	377.8	90	41.25789	792.6	42.73398		818.4	
42	19.25368	380.6	19.94252	401.3	394.3	386.1	91	41.71631	800.2	43.20880		829.2	
43	19.71210	389.4	20.41735	410.5	403.7	394.2	92	42.17473	810.8	43.68362		841.1	
44	20.17052	399.4	20.89217	420.2	415.6	404.9	93	42.63315	821.3	44.15845		856.4	
45	20.62894	407.8	21.36699	429.4	424.8	414.7	94	43.09157	831.2				
46	21.08737	416.2	21.84181	439.0	433.8	423.7	95	43.55000	842.8				
47	21.54579	423.2	22.31663	448.7	443.3	432.8	96	44.00842	861.3				
48	22.00421	431.2	22.79146	458.2	452.8	442.6							

As comparisons of the two sets of data in Figure 2 and the standard deviations show, the agreement among the results for individual runs was poorer in the case of the {10·0} crystals than for the (00·1) crystals. All of the {10·0} runs were characterized by irregularities in the rate of movement of the fringes past the reference point and by a small increase in the number of fringes in the field of view at the higher temperatures. It was originally believed that these inconsistencies were caused by small recrystallized grains on the top and bottom surfaces of the truncated pyramids. After the third run, these two surfaces on each of the crystals were very carefully relapped to remove small recrystallized grains and to reduce considerably residual strains. Examination of the crystals after the seventh test indicated that the efforts to prevent the formation of recrystallized grains were successful, but the dilatation tests were still characterized by the irregularities noted in the earlier runs. It is possible that the inconsistencies were caused by the presence of $ZrFe_2$ phase, but all three of these samples had nearly equal quantities of this impurity.

It appears that the most logical explanation for the poorer quality of the {10·0} data is related to the dimensions of the crystals themselves. As is evident from Table II, the areas of the bottom surfaces on the {10·0} crystals were smaller than the corresponding surfaces of the (00·1) crystals, but their heights were nearly the same. The recommended dimensions for interferometry samples are a base area of approximately 1 sq cm and a height of approximately 1 cm. For samples with a square base, the ratio of the side of a base to the height would be approximately 1 to 1. The (00·1) crystals had, approximately, this ratio, but the {10·0} crystals had ratios varying from 0.60:1 to 0.68:1. Apparently, the areas of the bases were too small for the height of the specimens. This could tend to make the samples unstable.

Any differential in the lateral expansion of the two optical flats would cause one or two of the specimens to tilt if the flats did not slide easily across the faces of the crystals. There is reason to believe that this is what happened in the tests with the {10·0} samples. At various points during the heating, one or two of the specimens became very slightly tilted. In effect, these samples were then longer than the ones which were not tilted, and they caused a small increase in the number of fringes in the field of view. When the optical flat slipped with respect to the ends of the samples, the number of fringes returned to the original value. At the temperatures near the $\alpha \rightarrow \beta$ transformation the optical flat did not slide across the surface of the sample at all, and the number of fringes in the field of view steadily increased. This caused an apparent slight decrease in the expansion rate, as shown in Figure 2, since an increase in the number of fringes resulted in an apparent decrease in the rate of expansion. In spite of these irregularities, the {10·0} data are believed to be reasonably good.

B. Comparison of Dilatation Data with Previous Results

In Table V are tabulated the coefficients for polynomial equations which represent various dimensions or properties of the alpha-zirconium lattice as functions of temperature. Included in the table are coefficients

Table V

COEFFICIENTS FOR POLYNOMIAL EQUATIONS WHICH EXPRESS DIMENSION OR PROPERTIES OF THE ALPHA-ZIRCONIUM LATTICE AS FUNCTIONS OF TEMPERATURE

Dimension or Property of the Alpha Zirconium Lattice	Coefficients Reported by Russell			Coefficients Derived from Equations Reported by Russell			Coefficients Obtained by Least-mean-square Analysis of Russell's Lattice-constant Data			Coefficients Obtained by Least-mean-square Analysis of Dilatation Data, or Derived from Said Coefficients and Lichter's[7] Lattice Constants at 25°C				
	k_0^*	$k_1 \times 10^6$	$k_2 \times 10^9$	k_0	$k_1 \times 10^6$	$k_2 \times 10^9$	k_0	$k_1 \times 10^6$	$k_2 \times 10^9$	k_0	$k_1 \times 10^6$	$k_2 \times 10^9$	$k_3 \times 10^{12}$	$k_4 \times 10^{15}$
a	3.23078	18.089	3.694				3.23115	11.990	14.538	3.23118	16.626	-	-	-
c	5.14691	31.415	36.416				5.14768	18.600	58.896	5.14634	47.413	-32.859	95.161	-50.722
$\frac{V}{atom} = \frac{\sqrt{3}}{4} a^2 c$				23.2628	402.658	221.253				23.2659	459.638	-163.392	445.449	-230.095
$\frac{c}{a}$				1.59308	0.864	9.313	1.59314	-0.150	11.014	1.59271	6.283	-9.628	28.981	-15.698
$\left(\frac{L_t}{L_0}\right)_{(\perp \text{ to c axis})} = \frac{a_t}{a_0}$				1.00000	5.599	1.143				1.00000	5.145	-	-	-
$\left(\frac{L_t}{L_0}\right)_{(\parallel \text{ to c axis})} = \frac{c_t}{c_0}$				1.00000	6.104	7.076				1.00000	9.213	-6.385	18.491	-9.856
$\frac{V_t}{V_0} = \frac{\sqrt{3}}{2} \left(\frac{a_t}{a_0}\right)^2 \left(\frac{c_t}{c_0}\right)$				1.00000	17.309	9.511				1.00000	19.756	-7.023	19.146	-9.890
$\frac{1}{a} \frac{da}{dt}$	5.599×10^{-6}	2.241×10^{-3}	-	5.630×10^{-6}	2.146×10^{-3}	-				5.136×10^{-6}	-	-	-	-
$\frac{1}{c} \frac{dc}{dt}$	6.106×10^{-6}	13.98×10^{-3}	-	6.173×10^{-6}	13.826×10^{-3}	-				7.921×10^{-6}	3.221×10^{-3}	7.465×10^{-3}	-	-
$\frac{1}{V/atom} \frac{dV/atom}{dt}$				17.490×10^{-6}	17.999×10^{-3}	-				17.116×10^{-6}	9.581×10^{-3}	-	-	-

* The coefficients k_i are for equations of the type: $Y_t = k_0 + k_1 t + \ldots\ldots k_p t^p$, where t is temperature expressed in °C, Y_t is the dimension or property, and p is the order of the polynomial.

reported by Russell,[1] coefficients recalculated from his lattice-constant data, and coefficients determined from the dilatation data. In calculating the various lattice dimensions as functions of temperature from the dilatation data, Lichter's values for "oxygen free" zirconium at 25°C were used as the reference lattice constants; these were[7]

$$a = 3.2316\ \text{Å} \quad ; \quad c = 5.1475\ \text{Å} \quad .$$

Experimental data for the lattice constants of alpha zirconium at various temperatures as reported by Russell[1] and by Skinner and Johnston[2] are tabulated in Table VI.

Table VI

LATTICE CONSTANTS, AXIAL RATIO, AND VOLUME PER ATOM FOR ALPHA ZIRCONIUM

Temperature (°C)	a (Å)	c (Å)	$\frac{c}{a}$	Volume per Atom ($Å^3$)
		Russell[1]		
10.0	3.23114	5.14741	1.59306	23.2703
17.2	3.23104	5.14733	1.59309	23.2684
17.5	3.23144	5.14807	1.59312	23.2776
19.0	3.23178	5.14831	1.59303	23.2836
97.4	3.23199	5.14970	1.59336	23.2928
144.4	3.23371	5.15291	1.59350	23.3322
185.5	3.23405	5.15377	1.59359	23.3410
255.7	3.23494	5.15669	1.59406	23.3671
326.0	3.23653	5.15955	1.59416	23.4030
377.9	3.23783	5.16372	1.59481	23.4408
421.9	3.23917	5.16588	1.59482	23.4699
470.2	3.24018	5.16856	1.59514	23.4967
504.5	3.24077	5.17118	1.59566	23.5173
523.7	3.23966	5.17144	1.59629	23.5023
563.7	3.24443	5.17712	1.59570	23.5974
579.0	3.24239	5.18071	1.59781	23.5841
		Skinner and Johnston[2]*		
25.0	3.2332	5.1477	1.5921	23.301
677.0	3.2460	5.1848	1.5973	23.655
759.0	3.2460	5.1858	1.5976	23.660
769.0	3.2473	5.1923	1.5990	23.708
846.0	3.2467	5.1910	1.5988	23.694
891.0	3.2480	5.1963	1.5998	23.737

*Skinner and Johnston's data were originally given in kX units. These have been converted to Å by multiplying by the factor 1.002063.

In the following comparisons of the dilatation results with previously reported results, greater significance is placed on the work of Russell. The work of Skinner and Johnston was considered less reliable because 1) they reported fewer observations, 2) they worked with zirconium which contained a significant quantity of hafnium, an alloying element which Russell has shown to have an effect on the thermal expansion of zirconium, and 3) they had to apply a correction to compensate for chemistry changes which occurred during the tests.

1. Thermal Expansion. The thermal expansion, or dimensional change per unit dimension at 0°C as a function of temperature, of the directions perpendicular and parallel to the c axis and of the volume are plotted in Figures 3 and 4. Curves are given for the dilatation data, the equations reported by Russell, and the average coefficients of thermal expansion (α_m) between 25° and 870°C given by Skinner and Johnston. These coefficients were

$$\alpha_{m(\perp \text{ to c axis})} = 5.5 \times 10^{-6} \text{ deg}^{-1}$$

and

$$\alpha_{m(\parallel \text{ to c axis})} = 10.8 \times 10^{-6} \text{ deg}^{-1} \quad .$$

In general, the discrepancies among the three sets of data are not excessively large. For the linear expansion parallel to the c axis, the dilatation data agreed reasonably well with Russell's data, although a greater expansion is indicated between 0 and about 300°C, and a lesser expansion between about 500 and 580°C. On the other hand, Skinner and Johnston's values of expansion are greater up to approximately 800°C, where they agree well with the dilatation data. Above that temperature their values were lower than the dilatation results.

The dilatation data for the linear expansion perpendicular to the c axis fall below the results of Russell and those of Skinner and Johnston. Russell's value at 580°C, the maximum temperature of his lattice-constant studies, was about 22% greater than that given by the dilatation data, whereas Skinner and Johnston's value was only 7% greater. Possible explanations for these differences are given in the section dealing with lattice constants as functions of temperature.

Table VII gives the coefficients of the polynomial equations for calculating the linear thermal expansion in a direction inclined θ degrees to the c axis according to equation 5. Coefficients are given for 5° intervals of the angle θ.

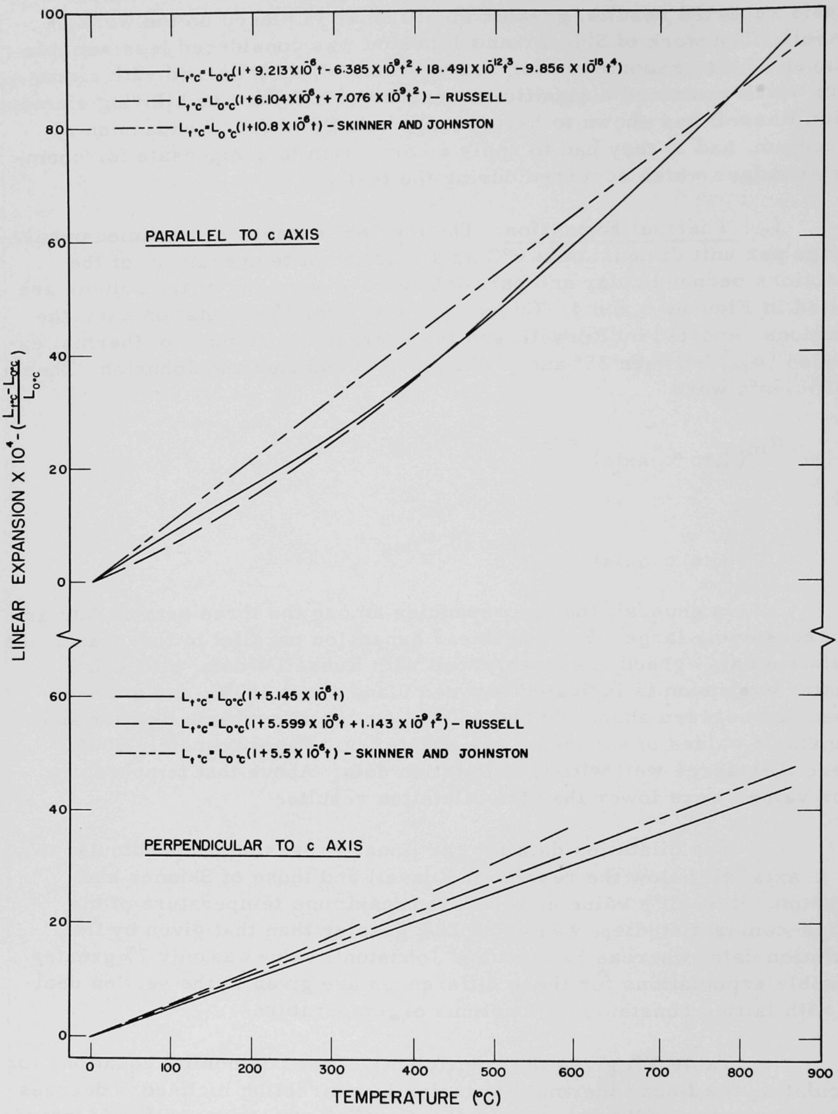

Fig. 3. Linear Thermal Expansion of Alpha Zirconium

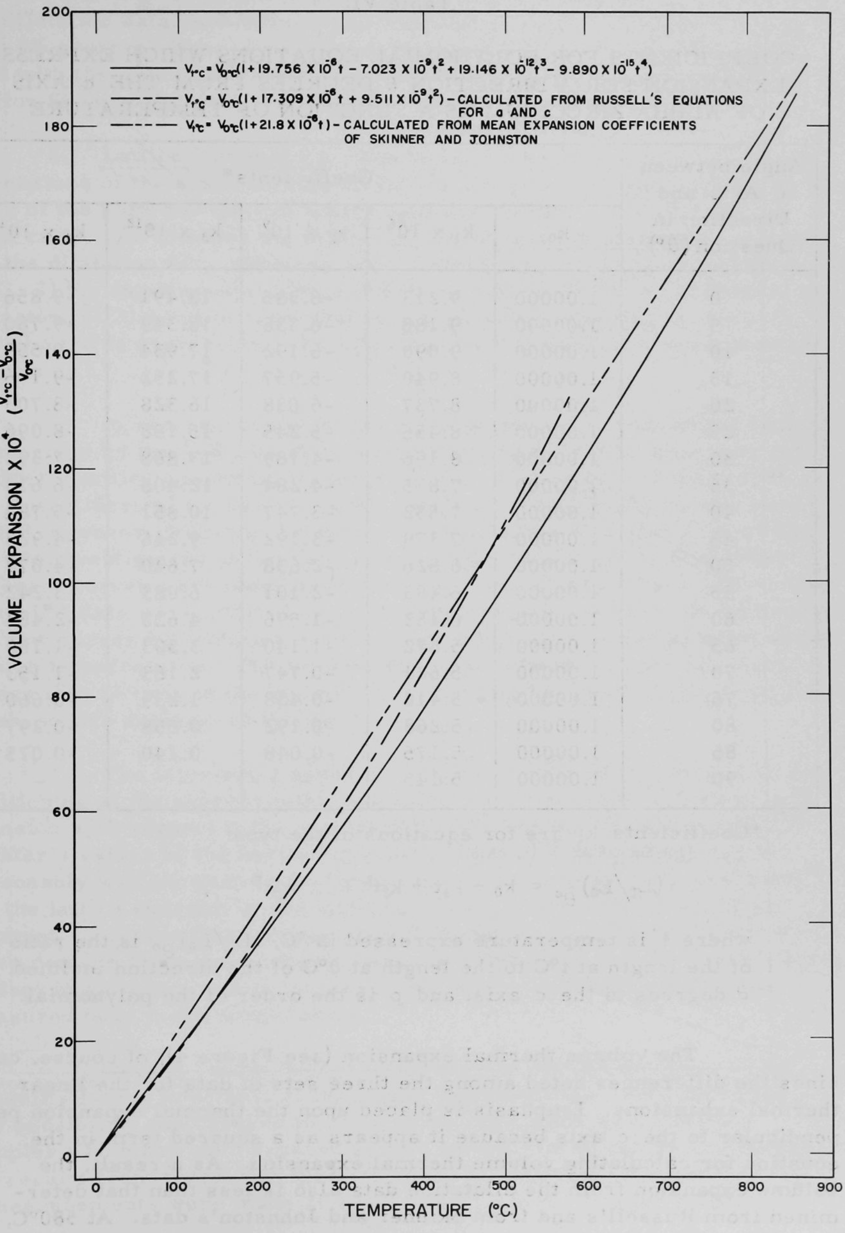

Fig. 4. Volume Thermal Expansion of Alpha Zirconium

Table VII

COEFFICIENTS FOR POLYNOMIAL EQUATIONS WHICH EXPRESS EXPANSION FOR A DIRECTION θ DEGREES FROM THE c AXIS OF ALPHA ZIRCONIUM AS A FUNCTION OF TEMPERATURE

Angle between c Axis and Direction in Question (θ°)	Coefficients*				
	k_0	$k_1 \times 10^6$	$k_2 \times 10^9$	$k_3 \times 10^{12}$	$k_4 \times 10^{15}$
0	1.00000	9.213	-6.385	18.491	-9.856
5	1.00000	9.185	-6.336	18.349	-9.780
10	1.00000	9.090	-6.192	17.934	-9.559
15	1.00000	8.940	-5.957	17.252	-9.196
20	1.00000	8.737	-5.638	16.328	-8.703
25	1.00000	8.456	-5.245	15.188	-8.096
30	1.00000	8.196	-4.789	13.868	-7.392
35	1.00000	7.875	-4.284	12.408	-6.613
40	1.00000	7.532	-3.747	10.851	-5.784
45	1.00000	7.179	-3.192	9.246	-4.928
50	1.00000	6.826	-2.638	7.640	-4.072
55	1.00000	6.483	-2.101	6.083	-3.242
60	1.00000	6.162	-1.596	4.623	-2.464
65	1.00000	5.872	-1.140	3.303	-1.760
70	1.00000	5.621	-0.747	2.163	-1.153
75	1.00000	5.418	-0.428	1.239	-0.660
80	1.00000	5.268	-0.192	0.558	-0.297
85	1.00000	5.175	-0.048	0.140	-0.075
90	1.00000	5.145	-	-	-

*Coefficients k_i are for equations of the type:

$$(L_t/L_0)_{\theta°} = k_0 + k_1 t + k_2 t^2 + \ldots k_p t^p \quad ,$$

where t is temperature expressed in °C, $(L_t/L_0)_{\theta°}$ is the ratio of the length at t°C to the length at 0°C of the direction inclined θ degrees to the c axis, and p is the order of the polynomial.

The volume thermal expansion (see Figure 4), of course, combines the differences noted among the three sets of data for the linear thermal expansions. Emphasis is placed upon the thermal expansion perpendicular to the c axis because it appears as a squared term in the equation for calculating volume thermal expansion. As a result, the volume expansion from the dilatation data also is less than that determined from Russell's and from Skinner and Johnston's data. At 580°C, Russell indicated a volume expansion 13% greater than that obtained from the dilatation data, and Skinner and Johnston's results indicated an expansion 8% greater. Near the lower temperatures Russell's results agree well with the dilatation data, and at temperatures near the phase transformation Skinner and Johnston's results agree reasonably well.

2. Lattice Dimensions. The temperature dependence of various dimensions of the alpha-zirconium lattice are given in Figures 5 through 8. Each of the plots for the two lattice constants a and c and that for the axial ratio, c/a includes the following information: 1) a curve representing the dilatation data combined with Lichter's values of the constants at 25°C, 2) the experimental lattice constants of Russell(1) and of Skinner and Johnston,(2) 3) a curve representing the equations reported by Russell, and 4) a curve representing the equation fitted to Russell's experimental results.

In general, comparisons of the a and c curves versus temperature given by Russell with those calculated from the dilatation data and Lichter's lattice constants, parallel similar comparisons of the curves for linear thermal expansion. Some relative shifting of the curves resulted, however, from the differences in the values of a and c given by the two investigators at the reference temperature of 25°C. Somewhat different curves were obtained for both the lattice constants a and c when Russell's data were refitted by least-mean-square analyses. In both cases, the new curves had smaller slopes at low temperatures and greater slopes at high temperatures. The reasons for these differences are not clear, unless they pertain to the methods of computing the polynomials from least-mean-square analysis of the data.

The interesting aspects of the plots of a and c pertain to the positioning of the experimental data points of Russell and of Skinner and Johnston with respect to the curves derived from the dilatation data and Lichter's values of the lattice constants. Russell's data points agree reasonably well between room temperature and up to approximately 400°C for the lattice constant a and up to approximately 500°C for the lattice constant c. Above these temperatures, Russell's values are greater than those derived from the dilatation data. Lichter found that increased oxygen content in alpha zirconium increased the lattice constants which were measured near room temperature.

Oxygen contamination could account for the increase in the Russell's lattice constants at the higher temperatures. He considered this possibility and checked for oxygen contamination by measuring samples which had been exposed at high temperatures and by comparing the results from these with those from fresh samples. Presumably the remeasurements were made at room temperature. He stated that no

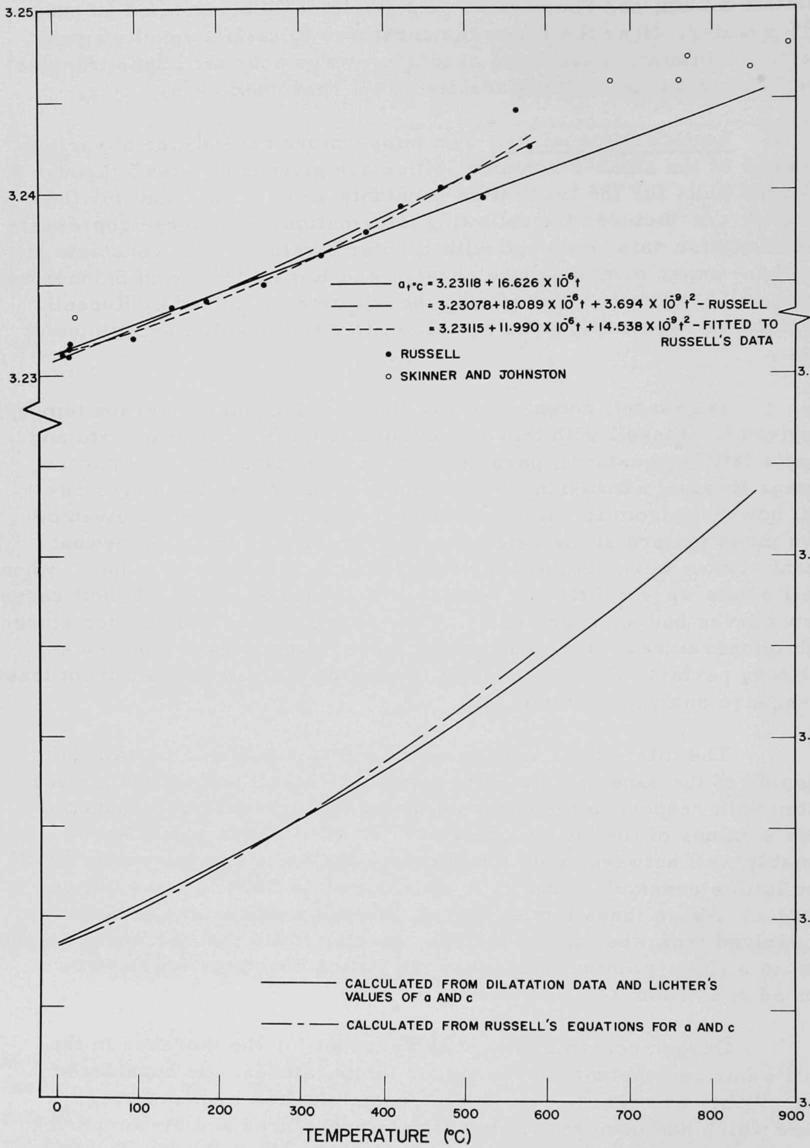

Fig. 5. Temperature Dependence of the Distance of Closest Approach and of the Lattice Constant a for Alpha Zirconium

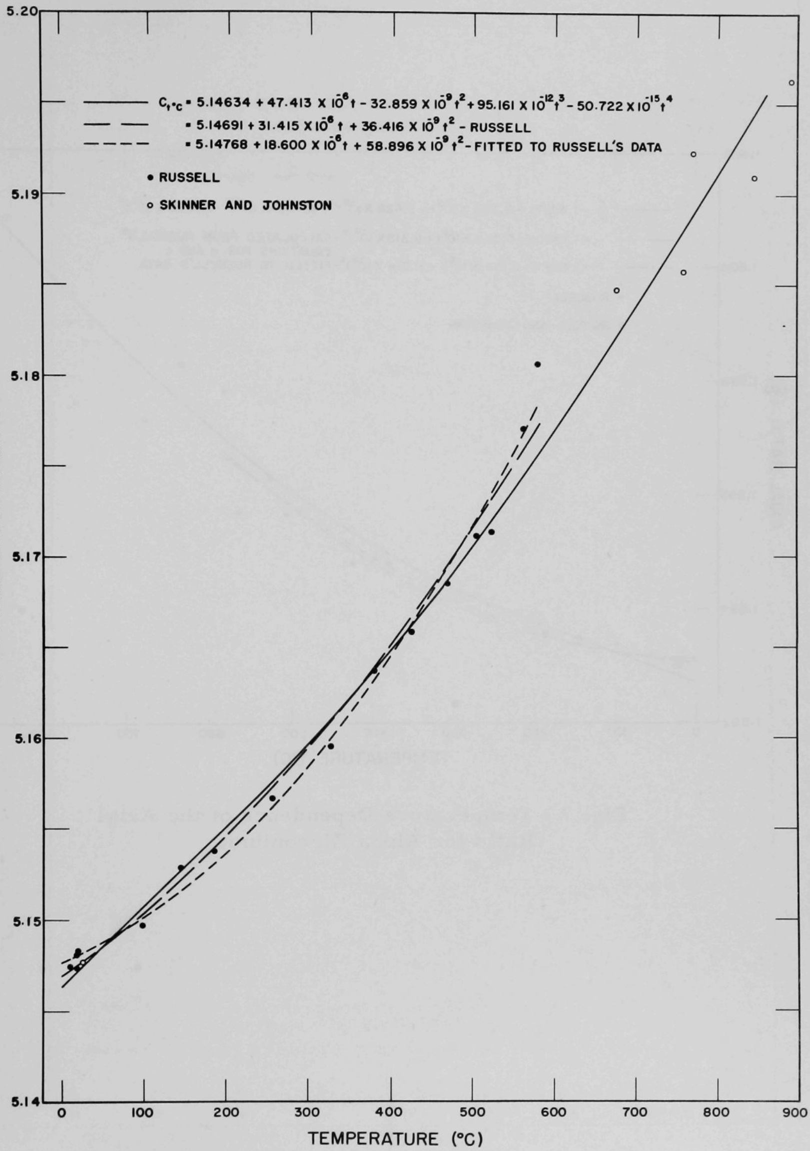

Fig. 6. Temperature Dependence of the Lattice Constant for Alpha Zirconium

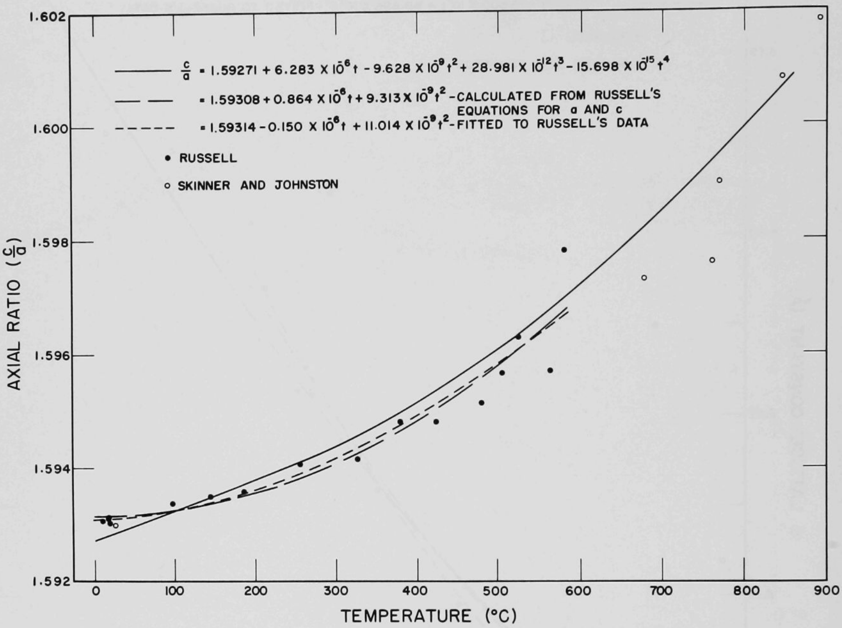

Fig. 7. Temperature Dependence of the Axial Ratio for Alpha Zirconium

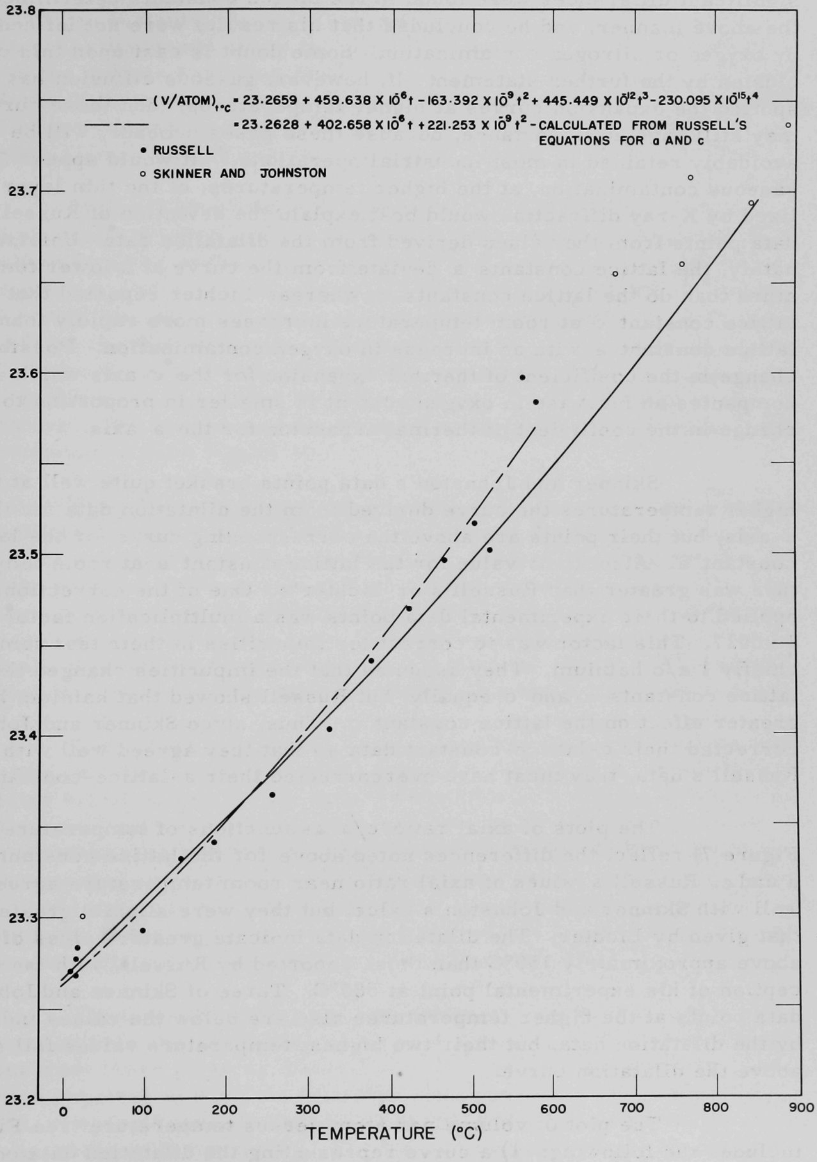

Fig. 8. Temperature Dependence of the Volume per Atom for Alpha Zirconium

significant differences were found in the lattice constants determined in the above manner, and he concluded that his results were not influenced by oxygen or nitrogen contamination. Some doubt is cast upon this conclusion by the further statement "If, however, gaseous diffusion has spoiled the expansion curves at higher temperatures, then these curves may still have an importance, because these gases probably will be unavoidably retained in most industrial operations." It would appear that gaseous contamination, at the higher temperatures, of the thin layer utilized by X-ray diffraction would best explain the deviation of Russell's data points from the values derived from the dilatation data. Unfortunately, the lattice constants a deviate from the curve at a lower temperature than do the lattice constants c, whereas Lichter reported that the lattice constant c at room temperature increases more rapidly than the lattice constant a with an increase in oxygen contamination. Possibly the change in the coefficient of thermal expansion for the c axis which accompanies an increase in oxygen content is smaller in proportion to the change in the coefficient of thermal expansion for the a axis.

Skinner and Johnston's data points bracket quite well at the higher temperatures the curve derived from the dilatation data for the c axis, but their points are above the corresponding curve for the lattice constant a. Also, their value for the lattice constant a at room temperature was greater than Russell's or Lichter's. One of the corrections applied to their experimental data points was a multiplication factor of 1.00027. This factor was to correct for impurities in their test samples, chiefly 1 a/o hafnium. They assumed that the impurities changed the lattice constants a and c equally, but Russell showed that hafnium had a greater effect on the lattice constant c. Thus, since Skinner and Johnston corrected their c-lattice-constant data so that they agreed well with Russell's data, they must have overcorrected their a-lattice-constant data.

The plots of axial ratio c/a as functions of temperature (see Figure 7) reflect the differences noted above for the lattice constants a and c. Russell's values of axial ratio near room temperature agreed well with Skinner and Johnston's value, but they were slightly greater than that given by Lichter. The dilatation data indicate greater values of c/a above approximately 150°C than those reported by Russell, with the exception of his experimental point at 580°C. Three of Skinner and Johnston's data points at the higher temperatures also are below the values indicated by the dilatation data, but their two highest temperature values fall slightly above the dilatation curve.

The plot of volume per atom versus temperature (see Figure 8) includes the following: 1) a curve representing the dilatation data combined with Lichter's values of the lattice constants at 25°C, 2) values calculated from the lattice constants reported by Russell and by Skinner and

Johnston, and 3) a curve calculated from the equations given by Russell for the lattice constants. Again, a comparison of the various data reflects the differences discussed above for the lattice constants a and c. Russell's values of the volume per atom agree well with the curve derived from the dilatation data up to approximately 400°C. Above that temperature Russell's data points are above the dilatation curve. All of Skinner and Johnston's data points also are above the curve from the dilatation data.

Figure 5 gives the temperature variation of the distance of closest approach in alpha zirconium as calculated from the dilatation data and Lichter's values of the lattice constants and as calculated from the equations for a and c which were reported by Russell. Again, the values obtained from Russell's equations are not appreciably different from those derived from the dilatation data. His curve is slightly below the dilatation curve between 0 and approximately 300°C. Between 300 and 580°C it is above the dilatation curve, and the divergence is greater the higher the temperature. The second nearest distance of approach corresponds to the lattice constant a (see Figure 5).

3. <u>True Thermal Expansion Coefficients</u>. True thermal expansion coefficients, which are defined by equations of the type given above (equation 8), are plotted in Figures 9 and 10. Each of the plots for the directions parallel to the c axis and perpendicular to the c axis includes: 1) a curve derived from the dilatation data and Lichter's values of the lattice constants at 25°C, 2) a curve representing the equation reported by Russell, and 3) a curve derived from the equation which was reported by Russell for the respective lattice constant as a function of temperature.

The curves based upon Russell's values of a and c agree well despite the fact they were obtained in two different manners. Russell differentiated his equations for the lattice constants as functions of temperature at specific values of temperature and then he fit equations to those values, whereas the equations for the second set of curves were obtained by differentiating equations fit to the logarithm of the lattice constant as functions of temperature. The values of the lattice constants used in evaluating the latter equations were those calculated from Russell's equations for the lattice constants as functions of temperature.

The true linear thermal expansion coefficients derived from the dilatation data and Lichter's values of a and c at 25°C are somewhat different from those given by Russell (see Figure 9). The coefficient for the lattice constant a from the dilatation data does not depend upon temperature, whereas Russell's data indicate that the coefficient increases with an increase in temperature. For the lattice constant c, the dilatation data do show a temperature dependence of the true linear thermal expansion coefficient, and the slope of the curve increases as the temperature increases. The values from the dilatation data are greater than

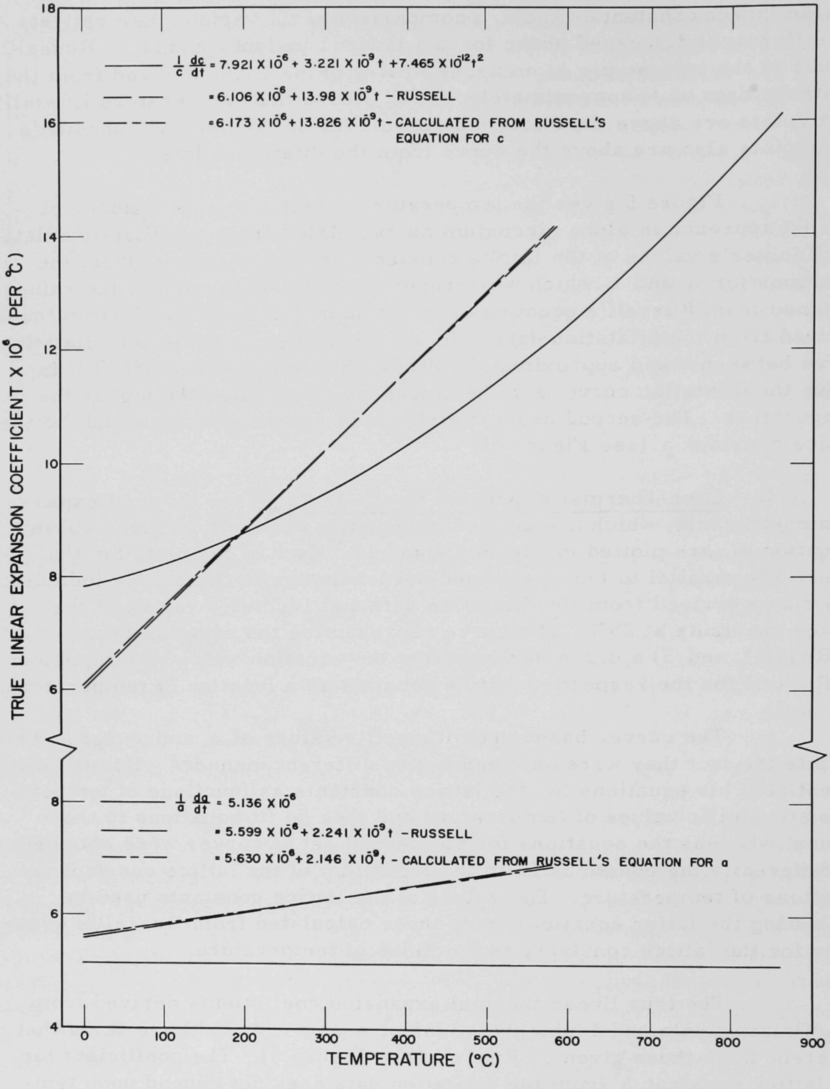

Fig. 9. True Linear Thermal Expansion Coefficients of Alpha Zirconium

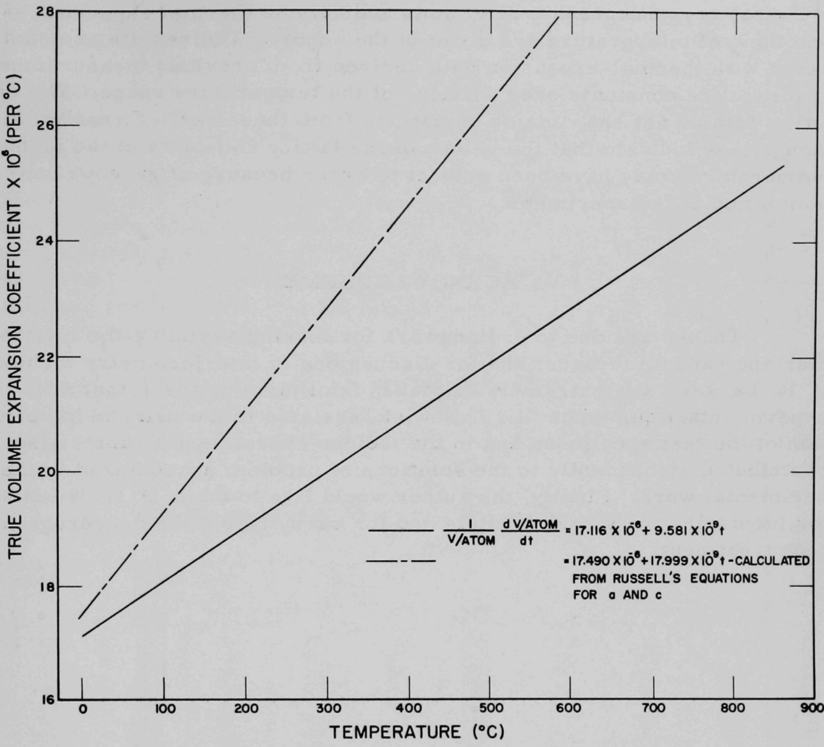

Fig. 10. True Volume Thermal Expansion Coefficients of Alpha Zirconium

those reported by Russell between room temperature and approximately 200°C. Above 200°C the values given by Russell are greater. Thus, at the higher temperatures, Russell's data showed larger values of the true linear thermal expansion coefficients for both the lattice constants a and c. The curves of true volume thermal expansion coefficient, given in Figure 10, reflect the larger numerical values given by Russell for the linear coefficients of the lattice constants a and c.

IV. CONCLUSIONS

Dilatation interferometry has been used to measure the thermal expansion of single crystals of alpha zirconium between room temperature and 860°C. Equations which represent linear thermal expansion in the two principal crystallographic directions and volume thermal expansion as functions of temperature are given in the report. The results are compared with thermal-expansion data derived from previous measurements of the lattice constants over portions of the temperature range. The dilatation data do not show major deviations from these earlier results, although they indicate that the values of the lattice constants at the higher temperatures may have been subject to error because of gaseous contamination of the specimens.

V. ACKNOWLEDGEMENTS

Thanks are due to J. Handwerk for making available the interferometer and vacuum furnace, and for discussions of interferometry techniques. D. R. Lankard was extremely helpful in familiarizing the author with the experimental equipment. L. J. Nowicki assisted in the painstaking preparation of the test specimens and in the tedious experimental runs. Also, he contributed significantly to the solutions of problem encountered in the experimental work. Finally, the author would like to thank D. G. Westlake for furnishing the single crystals and for carrying out the dehydrogenation heat treatment.

VI. APPENDIX

CORRECTION OF OBSERVED THERMOCOUPLE MILLIVOLTAGE

Since the interferometer was used under dynamic conditions, that is, continuous increase in temperature with time, it was necessary to determine the inherent differential between the observed temperature and the true temperature of the test specimens for the experimental conditions of the tests. For this purpose some calibration runs were made with a platinum specimen as a standard. The same physical arrangement of the interferometer assembly was used for the calibration runs and the experimental runs. Particular care was exercised when removing one set of specimens and inserting another set so as not to move the lower optical flat nor the thermocouple bead.

Table VIII gives the times required to heat the interferometer assembly to each whole integer of millivoltage output from a chromel-alumel thermocouple for each of the runs whose data were used in the

Table VIII

TRUE TEMPERATURE OF THE SPECIMENS AS A FUNCTION OF TIME FOR THE DILATATION RUNS

True Specimen Temperature (°C)	Elapsed Time (min)								
	Platinum Standard		(00·1) Zirconium Crystals		{10·0} Zirconium Crystals				Average
	Run No. 2	Run No. 3	Run No. 6	Run No. 7	Run No. 2	Run No. 5	Run No. 6	Run No. 7	
22.1	20.7	15.4	20.0	17.5	25.7	15.2	14.6	16.5	18.2
39.9	52.6	42.1	46.6	44.3	53.1	40.9	37.0	41.8	44.8
62.1	72.8	64.4	68.2	65.2	72.6	62.0	57.0	61.6	65.6
86.5	87.1	80.9	83.9	81.3	86.7	78.5	74.5	77.5	81.3
110.6	98.4	94.2	96.3	94.0	98.0	91.6	88.2	90.2	93.7
136.7	108.1	105.1	106.7	104.8	107.7	102.5	100.0	101.2	104.5
163.2	116.7	114.4	115.9	114.2	116.3	112.2	110.1	110.9	113.8
189.6	124.1	122.6	124.0	122.7	124.2	120.8	119.2	119.6	122.2
215.7	131.2	130.0	131.4	130.5	131.7	128.8	127.4	127.6	129.8
241.8	137.9	137.1	138.4	137.8	138.7	136.1	135.1	135.1	137.0
267.6	144.1	143.8	145.0	144.6	145.4	143.0	142.3	142.3	143.8
293.0	150.2	150.3	151.4	151.1	151.9	149.7	149.3	149.2	150.4
318.0	156.1	156.6	157.5	157.5	158.0	156.0	155.9	155.9	156.6
343.0	161.9	162.7	163.6	163.6	164.1	162.1	162.3	162.3	162.8
367.2	167.6	168.8	169.6	169.8	170.2	168.2	168.5	168.8	168.9
396.0	173.3	174.8	175.4	175.7	176.1	174.2	174.7	175.0	174.9
414.9	179.0	180.7	181.1	181.5	181.9	180.1	180.8	181.4	180.8
438.5	184.7	186.6	186.8	187.2	187.5	186.0	186.8	187.5	186.6
462.3	190.3	192.5	192.4	192.9	193.0	191.8	192.4	193.6	192.4
485.7	195.9	198.3	198.0	198.5	198.4	197.5	198.1	199.5	198.0
509.2	201.6	204.1	203.6	204.1	203.9	203.2	203.8	205.4	203.7
532.6	207.3	210.0	209.2	210.0	209.3	209.0	209.5	211.3	209.5
556.0	213.0	215.9	215.1	215.9	214.8	214.8	215.3	217.2	215.3
579.4	218.7	221.7	221.1	221.9	220.2	220.4	221.3	222.9	221.3
603.8	224.3	227.3	226.9	227.8	225.8	226.0	227.1	228.4	226.7
626.2	229.6	232.8	232.4	233.4	231.2	231.2	232.7	233.8	232.1
649.8	234.8	238.1	238.0	238.8	236.6	236.5	238.1	239.0	236.9
672.9	240.2	243.6	243.7	244.2	242.2	241.9	243.7	244.4	243.0
696.3	245.9	249.3	249.5	249.7	248.0	247.5	249.4	250.0	248.7
720.2	251.4	255.0	255.3	255.1	253.8	253.2	255.2	255.8	254.4
744.2	257.0	260.4	261.0	260.3	259.4	258.7	260.9	261.3	259.9
768.4	262.3	265.5	266.2	265.2	264.6	264.0	266.2	266.6	265.1
793.0	267.3	270.2	271.0	270.0	269.5	268.9	271.2	271.4	269.9
817.7	272.0	274.8	275.5	274.5	274.0	273.5	275.9	275.9	274.5
842.6	276.3	279.1	279.8	278.9	278.4	277.9	280.3	280.1	278.9
867.7	280.9	283.9	-	-	-	-	-	-	282.4

present report. The millivoltage values were corrected to represent true specimen temperature according to the procedure given below and converted to temperature in °C. In Figure 11 is plotted the true specimen temperatures from Table VIII as a function of the average elapsed times of the dilatation tests. Although the heating rate was not uniform over the entire temperature range, Table VIII shows that the rates were reproduced reasonably well from run to run. Consequently, temperature corrections based upon the calibration runs with platinum were applicable to the runs with the unknown specimens.

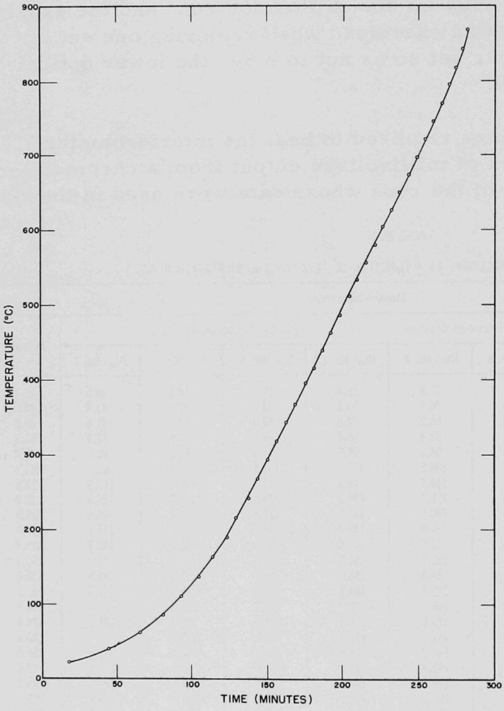

Fig. 11

True Temperature of the Specimens as a Function of Time for the Dilatation Runs

Three tests were made with the platinum standard specimen. The data from the first run were neglected on the assumption that "seating-in" occurred during the heating and cooling cycle. The observed temperatures, in terms of chromel-alumel thermocouple millivoltage, when the interference fringes coincided with the reference mark on the lower optical flat are given in Table IX for the second and third runs. The average of the two values for the coincidence of each fringe was compared with a corresponding thermocouple millivoltage which represented the true temperature of the platinum sample. The latter millivoltage was determined as follows.

Table IX

CORRECTION APPLIED TO THE OBSERVED THERMOCOUPLE MILLIVOLTAGE

Fringe No.	Observed Thermocouple emf (mv)			Calculated Thermocouple emf (mv)	Correction Calculated-Ave Observed (mv)
	Run No. 2	Run No. 3	Average		
0	0.560	0.560	0.560	0.620	+0.060
1	0.937	0.986	0.962	0.860	-0.102
2	1.294	1.415	1.355	1.108	-0.247
3	1.608	1.782	1.695	1.348	-0.347
4	1.897	2.095	1.996	1.602	-0.394
5	2.177	2.383	2.280	1.842	-0.438
6	2.449	2.663	2.556	2.096	-0.460
7	2.716	2.925	2.820	2.346	-0.474
8	-	3.180	-	2.600	-
9	3.242	3.440	3.341	2.850	-0.491
10	3.510	3.695	3.602	3.100	-0.502
11	3.769	3.950	3.860	3.350	-0.510
12	4.007	4.201	4.104	3.600	-0.504
13	4.250	4.447	4.348	3.850	-0.498
14	4.489	4.690	4.590	4.096	-0.494
15	4.727	4.907	4.817	4.346	-0.471
16	4.952	5.133	5.042	4.596	-0.446
17	5.180	5.345	5.262	4.832	-0.430
18	5.407	5.572	5.490	5.082	-0.408
19	5.632	5.807	5.720	5.318	-0.402
20	5.866	6.039	5.952	5.554	-0.398
21	6.097	6.283	6.190	5.790	-0.400
22	6.323	6.521	6.422	6.020	-0.402
23	6.548	6.720	6.634	6.259	-0.375
24	6.770	6.929	6.850	6.498	-0.352
25	6.985	7.171	7.078	6.734	-0.344
26	7.206	7.368	7.287	6.970	-0.317
27	7.424	-	-	7.202	-
28	7.672	7.807	7.740	7.438	-0.302
29	7.911	8.038	7.974	7.674	-0 300
30	8.138	8.262	8.200	7.906	-0.294
31	8.353	8.478	8.416	8.138	-0.278
32	8.575	8.702	8.640	8.374	-0.266
33	8.790	8.931	8.860	8.616	-0.244
34	9.002	9.141	9.072	8.848	-0.224
35	9.231	9.359	9.295	9.080	-0.212
36	9.449	9.575	9.512	9.312	-0.200
37	9.665	9.813	9.739	9.544	-0.195
38	9.879	10.041	9.960	9.782	-0.178
39	10.100	10.275	10.188	10.014	-0.174
40	10.315	10.497	10.406	10.256	-0.150
41	10.534	10.718	10.626	10.484	-0.142
42	10.755	10.943	10.849	10.726	-0.123
43	10.979	11.164	11.072	10.964	-0.108
44	11.217	11.391	11.304	11.192	-0.112
45	11.450	11.621	11.536	11.430	-0.106
46	11.676	11.836	11.756	11.662	-0.094
47	11.905	12.059	11.982	11.900	-0.082
48	12.131	12.285	12.208	12.138	-0.070
49	12.353	-	-	12.370	-
50	12.576	12.728	12.652	12.605	-0.047
51	12.800	12.953	12.876	12.840	-0.036
52	13.032	13.191	13.112	13.076	-0.036
53	-	13.422	-	13.310	-
54	13.481	13.655	13.568	13.548	-0.020
55	13.712	13.878	13.795	13.782	-0.013
56	13.951	14.102	14.026	14.016	-0.010
57	14.170	14.338	14.254	14.254	0.000
58	14.399	14.564	14.482	14.488	+0.006
59	14.629	14.795	14.712	14.725	+0.013
60	14.862	15.025	14.944	14.968	+0.024
61	15.083	15.261	15.172	15.200	+0.028
62	15.333	15.494	15.414	15.434	+0.020
63	15.562	15.729	15.646	15.664	+0.018
64	15.797	15.961	15.879	15.898	+0.019
65	-	16.300	-	16.132	-
66	16.266	16.436	16.351	16.376	+0.025
67	16.501	16.665	16.583	16.606	+0.023
68	16.730	16.892	16.811	16.836	+0.025
69	16.964	17.124	17.044	17.070	+0.026
70	17.194	17.351	17.272	17.310	+0.038
71	17.431	17.585	17.508	17.540	+0.032
72	17.666	17.820	17.743	17.770	+0.027
73	17.896	18.051	17.974	18.010	+0.036
74	18.125	18.283	18.204	18.240	+0.036
75	18.362	18.523	18.442	18.470	+0.028
76	18.599	18.751	18.675	18.705	+0.030
77	18.829	18.976	18.902	18.940	+0.038
78	19.055	19.212	19.134	19.166	+0.032
79	19.292	19.441	19.366	19.406	+0.040
80	19.518	19.677	19.598	19.632	+0.034
81	19.748	19.900	19.824	19.872	+0.048
82	19.984	20.135	20.060	20.098	+0.038
83	20.210	20.352	20.281	20.338	+0.057
84	20.448	20.582	20.515	20.564	+0.049
85	20.679	20.822	20.750	20.795	+0.045
86	20.904	21.054	20.979	21.026	+0.047
87	21.138	21.283	21.210	21.252	+0.042
88	21.369	21.411	21.390	21.478	+0.088
89	21.601	21.733	21.667	21.714	+0.047
90	21.842	21.964	21.903	21.945	+0.042
91	22.080	22.203	22.142	22.172	+0.030
92	22.309	22.429	22.369	22.398	+0.029
93	22.540	22.661	22.600	22.634	+0.034
94	22.775	22.886	22.830	22.856	+0.026
95	22.995	23.108	23.052	23.085	+0.033
96	23.230	23.332	23.281	23.314	+0.033
97	23.452	23.561	23.506	23.536	+0.030
98	23.685	23.803	23.744	23.768	+0.024
99	23.911	24.021	23.966	23.994	+0.028
100	24.132	24.253	24.192	24.226	+0.034
101	24.359	24.472	24.416	24.448	+0.032
102	24.590	24.711	24.650	24.675	+0.025
103	24.811	24.936	24.874	24.902	+0.028
104	25.028	25.167	25.098	25.120	+0.022
105	25.256	25.385	25.320	25.352	+0.032
106	25.480	25.597	25.538	25.574	+0.036
107	25.708	25.828	25.768	25.802	+0.034
108	25.932	26.053	25.992	26.024	+0.032
109	26.147	26.288	26.218	26.242	+0.024
110	26.372	26.500	26.436	26.464	+0.028
111	26.603	26.722	26.662	26.690	+0.028
112	26.834	26.954	26.894	26.910	+0.016
113	27.110	27.172	27.141	27.132	-0.009
114	27.279	27.398	27.338	27.350	+0.012
115	27.509	27.612	27.560	27.568	+0.008
116	27.729	27.822	27.776	27.786	+0.010
117	27.952	28.066	28.009	28.004	-0.005
118	28.182	28.295	28.238	28.218	-0.020
119	28.402	28.512	28.457	28.436	-0.021
120	28.619	28.726	28.672	28.654	-0.018
121	28.839	29.021	28.930	28.872	-0.058
122	29.062	29.173	29.118	29.096	-0.022
123	29.278	29.383	29.330	29.305	-0.025
124	29.490	29.600	29.545	29.528	-0.017
125	-	29.813	-	29.732	-
126	29.925	30.039	29.982	29.950	-0.032
127	30.127	30.260	30.194	30.164	-0.030
128	30.338	30.452	30.395	30.370	-0.025
129	30.556	30.663	30.610	30.596	-0.014
130	30.763	30.878	30.820	30.810	-0.010
131	30.977	31.083	31.030	31.024	-0.006
132	31.193	31.308	31.250	31.238	-0.012
133	31.407	31.503	31.455	31.448	-0.007
134	31.608	31.714	31.661	31.662	+0.001
135	31.836	31.930	31.883	31.866	-0.017
136	32.038	32.149	32.094	32.080	-0.014
137	32.239	32.342	32.290	32.290	0.000
138	32.452	32.550	32.501	32.504	+0.003
139	32.655	32.760	32.708	32.704	-0.004
140	32.858	32.957	32.908	32.918	+0.010
141	33.068	33.160	33.114	33.118	+0.008
142	33.257	33.359	33.308	33.328	+0.020
143	33.466	33.565	33.516	33.532	+0.016
144	33.669	33.765	33.717	33.742	+0.025
145	33.874	33.965	33.920	33.942	+0.022
146	34.075	34.172	34.124	34.152	+0.028
147	34.270	34.371	34.320	34.352	+0.032
148	34.470	34.569	34.520	34.562	+0.042
149	34.667	34.776	34.722	34.762	+0.042
150	34.864	34.971	34.918	34.962	+0.044
151	35.070	35.170	35.120	35.168	+0.048
152	35.277	35.364	35.320	35.368	+0.048
153	35.480	35.556	35.518	35.564	+0.046
154	35.675	35.752	35.714	35.774	+0.060
155	35.868	35.950	35.909	35.974	+0.065
156	36.069	36.153	36.111	36.170	+0.059
157	36.257	36.350	36.304	36.366	+0.062

The original length of the platinum sample was 0.4968 cm at 15.5°C, the starting temperature of the runs. When the n^{th} fringe passed the reference mark, the specimen had expanded to the length L_t at temperature t:

$$L_{t°C} = L_{15.5°C} + \frac{n\lambda}{2}$$

$$= L_{15.5°C} + \frac{0.5461 \times 10^{-4}}{2}$$

$$= L_{15.5°C} + 0.27305 \times 10^{-4} n \quad , \qquad (10)$$

where λ is the wavelength of the mercury vapor green light in cm.

The accepted equation for expansion of platinum between 0° and 1000°C is as follows:(8)

$$L_{t°C} = L_{0°C}(1 + 8.9877 \times 10^{-6}t + 1.0652 \times 10^{-9}t^2 + 0.1256 \times 10^{-12}t^3) \quad . \qquad (11)$$

From the known length of the sample at 15.5°C, the value for $L_{0°C}$ in equation 11 is

$$L_{0°C} = 0.496730673 \text{ cm} \quad .$$

Combining equations 10 and 11 and substituting the value for $L_{0°C}$,

$$L_{t°C} = 0.4968 + 0.27305 \times 10^{-4}n$$

$$= 0.496730673(1 + 8.9877 \times 10^{-6}t + 1.0652 \times 10^{-9}t^2 + 0.1256 \times 10^{-12}t^3) \quad ;$$

$$n = -2.5390 + 1.6350 \times 10^{-1}t + 1.9377 \times 10^{-5}t^2 + 2.2853 \times 10^{-9}t^3 \quad . \qquad (12)$$

From this equation it is possible to solve for t for a given value of n.

Solutions for t were obtained for n equal to integers 0 through 157 with the aid of program ANE-203, zeroes of a polynomial,(9) and the IBM-704 computer. The values of t were converted to millivolts with a standard conversion chart for chromel-alumel thermocouples, and these were accepted as the true specimen temperature (see Table IX).

A graphical aid for correcting the observed temperature to true specimen temperature was constructed from the data in Table IX by plotting the true temperature minus the observed temperature, in millivolts, against the observed thermocouple millivoltage and by drawing a smooth curve through the points (see Figure 12). The correction curve appeared to be valid. As would be expected, the greatest corrections had to be applied at the lower temperatures, especially for a vacuum furnace, because the primary mode of heat transfer was by conduction. The maximum difference between observed and true temperatures was approximately 12.5°C. As the temperature became higher, heating by radiation was more efficient and the

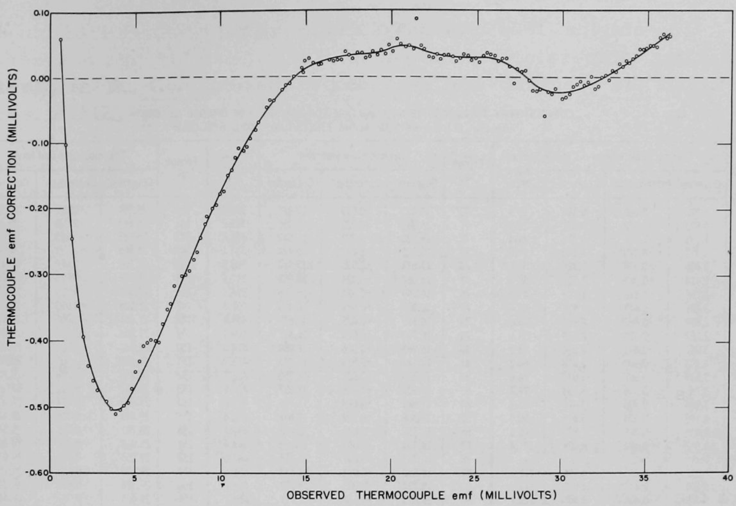

Fig. 12. Correction Applied to the Observed Thermocouple Millivoltages

difference between the true and observed temperatures became less. Above 14 mv (approximately 343°C), the applied correction amounted to 1°C or less. An example of the procedure for applying the correction to the observed thermocouple millivoltage is given in Table X.

Table X

CORRECTION OF THERMOCOUPLE MILLIVOLTAGE FOR RUN NO. 6 OF CHANGE IN LENGTH PARALLEL TO THE c AXIS OF ALPHA ZIRCONIUM - (00·1) SPECIMEN

Fringe No.	Thermocouple emf (mv)			Temperature (°C)	Fringe No.	Thermocouple emf (mv)			Temperature (°C)	Fringe No.	Thermocouple emf (mv)			Temperature (°C)
	Observed	Correction	Corrected			Observed	Correction	Corrected			Observed	Correction	Corrected	
0	0.592	+0.060	0.652	16.3	70	15.151	+0.019	15.170	370.8	140	26.539	+0.027	26.566	638.9
1	0.883	-0.072	0.811	20.3	71	15.331	+0.021	15.352	375.2	141	26.683	+0.026	26.709	642.2
2	1.153	-0.177	0.976	24.4	72	15.520	+0.022	15.542	379.8	142	26.823	+0.023	26.846	645.6
3	1.412	-0.270	1.142	28.6	73	15.702	+0.022	15.724	384.1	143	26.968	+0.021	26.989	649.0
4	1.666	-0.338	1.328	33.2	74	15.891	+0.023	15.914	388.6	144	27.108	+0.019	27.117	651.9
5	1.914	-0.383	1.531	38.0	75	16.062	+0.024	16.086	392.6	145	27.249	+0.016	27.265	655.6
6	2.158	-0.416	1.742	43.3	76	16.249	+0.024	16.273	397.1	146	27.386	+0.013	27.399	658.7
7	2.401	-0.443	1.958	48.4	77	16.429	+0.024	16.453	401.3	147	27.528	+0.010	27.538	662.0
8	2.629	-0.462	2.167	53.6	78	16.617	+0.025	16.642	405.8	148	27.666	+0.006	27.672	665.3
9	2.862	-0.489	2.373	58.6	79	16.792	+0.025	16.817	409.9	149	27.802	+0.003	27.805	668.4
10	3.087	-0.491	2.596	63.9	80	16.963	+0.026	16.989	414.0	150	27.947	-0.001	27.946	671.6
11	3.308	-0.500	2.808	69.2	81	17.147	+0.026	17.173	418.3	151	28.090	-0.005	28.085	675.1
12	3.539	-0.507	3.032	74.6	82	17.323	+0.027	17.350	422.5	152	28.230	-0.008	28.222	678.3
13	3.763	-0.510	3.253	79.8	83	17.509	+0.028	17.537	426.9	153	28.367	-0.011	28.356	681.4
14	3.984	-0.508	3.476	85.2	84	17.681	+0.028	17.709	431.0	154	28.508	-0.014	28.494	684.9
15	4.198	-0.504	3.694	90.4	85	17.868	+0.029	17.897	435.4	155	28.643	-0.016	28.627	687.9
16	4.426	-0.495	3.931	96.0	86	18.038	+0.030	18.068	439.4	156	28.781	-0.018	28.763	691.1
17	4.642	-0.484	4.158	101.4	87	18.208	+0.030	18.238	443.4	157	28.918	-0.019	28.899	694.4
18	-	-	-	-	88	18.385	+0.031	18.416	447.6	158	29.055	-0.020	29.035	697.6
19	5.057	-0.460	4.597	111.9	89	18.552	+0.032	18.584	451.6	159	29.190	-0.020	29.170	700.8
20	5.269	-0.447	4.822	117.6	90	18.724	+0.033	18.757	455.7	160	29.328	-0.020	29.308	704.2
21	5.479	-0.434	5.045	122.9	91	18.893	+0.034	18.927	459.7	161	29.465	-0.021	29.444	707.4
22	5.683	-0.422	5.261	128.3	92	19.068	+0.034	19.102	463.8	162	29.605	-0.021	29.584	710.6
23	5.894	-0.409	5.485	133.9	93	19.238	+0.035	19.273	467.8	163	29.742	-0.020	29.722	714.0
24	6.098	-0.396	5.702	139.3	94	19.405	+0.036	19.441	471.8	164	29.879	-0.020	29.859	717.2
25	6.298	-0.384	5.914	144.6	95	19.576	+0.037	19.613	475.8	165	30.016	-0.020	29.996	720.5
26	6.499	-0.373	6.126	149.9	96	19.748	+0.038	19.786	479.9	166	30.149	-0.020	30.129	723.7
27	6.700	-0.361	6.339	155.2	97	19.919	+0.039	19.958	484.0	167	30.284	-0.019	30.265	726.9
28	6.901	-0.350	6.551	160.5	98	20.079	+0.040	20.119	487.6	168	30.421	-0.018	30.403	730.3
29	7.099	-0.338	6.761	165.8	99	20.246	+0.040	20.286	491.6	169	30.550	-0.018	30.532	733.3
30	-	-	-	-	100	20.410	+0.040	20.450	495.4	170	30.690	-0.017	30.673	736.6
31	7.509	-0.315	7.194	176.6	101	20.569	+0.040	20.609	499.2	171	30.825	-0.016	30.809	740.0
32	7.706	-0.318	7.388	181.4	102	20.744	+0.040	20.784	503.3	172	30.962	-0.015	30.947	743.2
33	7.912	-0.292	7.620	187.2	103	20.908	+0.039	20.947	507.1	173	31.093	-0.014	31.079	746.5
34	8.114	-0.282	7.832	192.6	104	21.072	+0.038	21.110	511.0	174	31.227	-0.012	31.215	749.6
35	8.320	-0.270	8.050	198.0	105	21.235	+0.038	21.273	514.8	175	31.363	-0.011	31.352	753.0
36	8.519	-0.258	8.261	203.3	106	21.399	+0.037	21.436	518.6	176	31.499	-0.010	31.489	756.2
37	8.729	-0.247	8.482	208.6	107	21.551	+0.036	21.587	522.1	177	31.636	-0.008	31.628	759.6
38	8.928	-0.235	8.693	213.8	108	21.716	+0.035	21.751	526.0	178	31.768	-0.007	31.761	762.8
39	9.136	-0.224	8.912	219.3	109	21.878	+0.035	21.913	529.8	179	31.899	-0.005	31.894	765.9
40	9.344	-0.212	9.132	224.8	110	22.039	+0.034	22.073	533.6	180	32.035	-0.004	32.031	769.3
41	9.541	-0.202	9.339	230.0	111	22.202	+0.033	22.235	537.4	181	32.164	-0.002	32.162	772.3
42	9.750	-0.191	9.559	235.5	112	22.361	+0.032	22.393	541.1	182	32.296	0.000	32.296	775.6
43	9.940	-0.181	9.759	240.2	113	22.519	+0.032	22.552	544.8	183	32.430	+0.002	32.432	779.0
44	10.140	-0.170	9.970	245.5	114	22.671	+0.031	22.702	548.3	184	32.560	+0.004	32.564	782.1
45	10.345	-0.159	10.186	250.6	115	22.826	+0.031	22.857	551.9	185	32.690	+0.006	32.696	785.4
46	10.540	-0.149	10.391	255.8	116	22.989	+0.030	23.019	555.7	186	32.814	+0.008	32.822	788.3
47	10.740	-0.139	10.601	260.8	117	23.140	+0.030	23.170	559.2	187	32.942	+0.010	32.952	791.5
48	10.938	-0.128	10.810	266.0	118	23.294	+0.030	23.324	562.8	188	33.079	+0.012	33.091	795.0
49	11.141	-0.118	11.023	271.1	119	23.445	+0.030	23.475	566.4	189	33.210	+0.014	33.224	798.1
50	11.329	-0.109	11.220	276.0	120	23.602	+0.030	23.632	570.0	190	33.329	+0.016	33.345	801.1
51	11.537	-0.099	11.438	281.2	121	23.752	+0.030	23.782	573.6	191	33.461	+0.018	33.479	804.4
52	11.730	-0.090	11.640	286.2	122	23.903	+0.030	23.933	577.1	192	33.597	+0.020	33.617	807.7
53	11.927	-0.081	11.846	291.2	123	24.051	+0.030	24.081	580.5	193	33.727	+0.022	33.749	811.0
54	-	-	-	-	124	24.203	+0.030	24.233	584.1	194	33.850	+0.024	33.874	814.1
55	12.313	-0.065	12.248	301.0	125	24.357	+0.030	24.387	587.7	195	33.979	+0.026	34.005	817.3
56	12.508	-0.057	12.451	305.8	126	24.503	+0.030	24.533	591.1	196	34.100	+0.028	34.128	820.2
57	12.699	-0.049	12.650	310.5	127	24.655	+0.030	24.685	594.7	197	34.231	+0.031	34.262	823.6
58	12.890	-0.042	12.848	315.4	128	24.800	+0.030	24.830	598.0	198	34.350	+0.033	34.383	826.6
59	13.081	-0.034	13.047	320.2	129	24.941	+0.030	24.971	601.4	199	34.482	+0.035	34.517	829.7
60	13.277	-0.028	13.249	325.0	130	25.084	+0.030	25.114	604.8	200	34.612	+0.038	34.650	833.0
61	13.466	-0.021	13.445	329.6	131	25.229	+0.030	25.259	608.2	201	34.743	+0.040	34.783	836.3
62	13.656	-0.014	13.642	334.6	132	25.381	+0.030	25.411	611.8	202	34.861	+0.042	34.903	839.3
63	13.840	-0.009	13.831	339.0	133	25.520	+0.030	25.550	615.0	203	34.989	+0.044	35.033	842.3
64	14.035	-0.004	14.031	343.8	134	25.668	+0.030	25.698	618.4	204	35.120	+0.046	35.166	845.6
65	14.217	+0.001	14.218	348.2	135	25.802	+0.030	25.832	621.6	205	35.241	+0.048	35.289	848.7
66	14.407	+0.006	14.413	352.8	136	25.953	+0.030	25.983	625.1	206	35.370	+0.050	35.420	852.0
67	14.586	+0.010	14.596	357.2	137	26.102	+0.030	26.132	628.8	207	35.498	+0.052	35.550	855.2
68	14.779	+0.014	14.793	361.8	138	26.239	+0.029	26.269	632.0	208	35.622	+0.054	35.676	858.2
69	14.965	+0.017	14.982	366.3	139	26.386	+0.028	26.414	635.4	209	35.749	+0.056	35.805	861.4

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